consideration. We illustrate their potential in characterizing and, where possible, identifying certain minimal structures. Further, while these methods are introduced in a purely topological setting, we show that they have a strong order-theoretic appeal. Their topological significance has a direct order-theoretic translation when we regard the space as a partially-ordered set.

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Abstract of Doctoral Thesis

DIMENSIONS OF COMMUTATIVE MATRIX ALGEBRAS

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Let F be a field and let $M_n(F)$ be the algebra of $n \times n$ matrices over F. Let $A, B \in M_n(F)$ with $AB = BA$ and let $\mathcal{A}$ be the algebra generated by A and B over F. A theorem of Gerstenhaber [Ann. Math. 73: 324–348 (1961)] states that the dimension of $\mathcal{A}$ is at most n. Gerstenhaber's proof uses the methods of algebraic geometry. In Chapter I of this thesis, we obtain a purely matrix–theoretic proof of the result, constructing in the process a basis for the algebra, $\mathcal{A}$. We also examine when equality occurs. The case where F is algebraically closed and $\mathcal{A}$ is indecomposable (under similarity) holds the key to the general situation. In this case, we obtain a Cayley–Hamilton–like theorem expressing B^k as a polynomial in $I, B, \ldots, B^{k-1}$ with coefficients in $F[A]$, where k denotes the number of blocks in the Jordan form of A. If all Jordan blocks of A have the same size, we say $\mathcal{A}$ is homogeneous. In this case we obtain a nonderogatory–like condition on B which is equivalent to $\dim_F \mathcal{A} = n$. We also show that in this case, $\dim_F \mathcal{A} = n$ is equivalent to the maximality of $\mathcal{A}$ as a commutative subalgebra of $M_n(F)$.

In Chapter II we examine the dimensions of three–generated commutative subalgebras of $M_n(F)$. Let A, B and $C \in M_n(F)$ be pairwise commutative, and let $\mathcal{A}$ be the algebra generated by A, B and C over F. It is an open question whether or not the dimension of $\mathcal{A}$ is bounded above by n. Again, the case where

F is algebraically closed and $\mathcal{A}$ is indecomposable holds the key concepts. If A, say, has r Jordan blocks, with the biggest Jordan block of size $k \times k$, then it is shown that generally $\dim_F \mathcal{A} \leq \{nk, kr(r+1)/2\}$. In the homogeneous case, it is shown that $\dim_F \mathcal{A} \leq n^{3/2}$, and if $\mathcal{A}$ has fewer than four Jordan blocks, then $\dim_F \mathcal{A} \leq n$. Further if the exponent of the algebra $\mathcal{A}$ is also k (i.e. $\mathcal{A}^k = 0$), then it is shown that for $n < 30$, $\dim_F \mathcal{A} \leq n$. In case $\mathcal{A}$ is homogeneous, then each matrix in $\mathcal{A}$ can be considered as an element of $M_r(F[J])$ (where $A = J \oplus \cdots \oplus J$, r blocks of $J = J_k$, the $k \times k$ Jordan block with associated eigenvalue zero). It is shown that if B is a Wasow matrix over the local commutative ring $F[J]$, i.e., B is similar over $F[J]$ to a matrix in rational canonical form, then again in this case the dimension of $\mathcal{A}$ cannot exceed n.

Let $\mathcal{A}$ be a commutative subalgebra of $M_n(F)$, and say the centralizer of $\mathcal{A}$, $\mathcal{C}(\mathcal{A})$, is contained in $\mathcal{A}$. Then $\mathcal{A}$ is said to be a maximal commutative subalgebra of $M_n(F)$. We define the exponent of $\mathcal{A}$ to be the smallest positive integer k such that $x_1 \ldots x_k = 0$ for all $x_1, \ldots x_k$ in the radical of $\mathcal{A}$. In Chapter III we study the dimensions of maximal commutative subalgebras of $M_n(F)$. A classical result of Schur states that $\dim_F \mathcal{A} \leq [1 + n^2/4]$, where [ ] denotes the greatest integer function. Courter [Duke Math. J. 32:225-232 (1965)] proved if $\mathcal{A}$ has exponent two then $\dim_F \mathcal{A} \geq n$. Laffey [Linear Alg. Appl. 71:199-212 (1985)] showed that generally $\dim_F \mathcal{A} \leq (2n)^{2/3} - 1$, and if $\mathcal{A}$ has exponent three then the best possible lower bound is $[3n^{2/3} - 4]$. We create a sequence of maximal commutative subalgebras $\mathcal{A}_n$, each with exponent four, with $\dim_F \mathcal{A}_n$ of the order of $n^{2/3} - n^{1/3}$, in the limit. On the other hand, if the exponent of $\mathcal{A}$ is greater than or equal to $n-3$, and the characteristic of F does not divide $n!$, then we show that $\dim_F \mathcal{A}$ is either n, $n+1$ or $n+2$.

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Research Announcement

PREDUALS OF SPACES OF HOLOMORPHIC FUNCTIONS

Christopher Boyd

For U an open subset of a locally convex space E we denote by $\mathcal{H}(U)$ the space of $\mathbb{C}$–valued holomorphic functions on U. In infinite dimensional holomorphy we consider three natural topologies on $\mathcal{H}(U)$. τ_o is the compact–open topology of convergence on compact subset of U. We say a semi–norm p is ported by the compact subset K of U if for each open set V, $K \subset V \subset U$, we can find $c(V) > 0$ such that $p(f) \leq c(V)\|f\|_V$ for every f in $\mathcal{H}(U)$. τ_ω is the topology generated by all semi–norms ported by compact subsets of U. Finally say that a semi–norm p is τ_δ continuous if for each countable increasing open cover $\{U_n\}_n$ of U there is $C > 0$ and $n_o \in \mathbb{N}$ such that $p(f) \leq C\|f\|_{U_{n_o}}$ for every $f \in \mathcal{H}(U)$. τ_δ is the topology on $\mathcal{H}(U)$ generated by all τ_δ continuous semi–norms. We always have

$$\tau_o \leq \tau_\omega \leq \tau_\delta$$

on $\mathcal{H}(U)$. $P(^nE)$ denotes the space of n-homogeneous polynomials on E. We note that τ_ω and τ_δ agree on $P(^nE)$ for every integer n. For K a compact subset of E we let $\mathcal{H}(K)$ denote the space of holomorphic germs on K. The τ_o(resp. τ_ω) topology on $\mathcal{H}(K)$ is defined by $(\mathcal{H}(K), \tau_o) = \text{ind}_{K \subset V}(\mathcal{H}(V), \tau_o)$ (resp. $(\mathcal{H}(K), \tau_\omega) = \text{ind}_{K \subset V}(\mathcal{H}(V), \tau_\omega)$).

Given a locally convex space E we let $E'_i = \text{ind}_V E'_{V^\circ}$, where the inductive limit is taken over all neighbourhoods V of 0 in E, and let E'_b denote the dual of E equipped with the topology of uniform convergence on bounded subsets of E.

In [3] Mujica and Nachbin shows there is a complete locally convex space $G(U)$ with the property that $G(U)'_i = (\mathcal{H}(U), \tau_\delta)$.