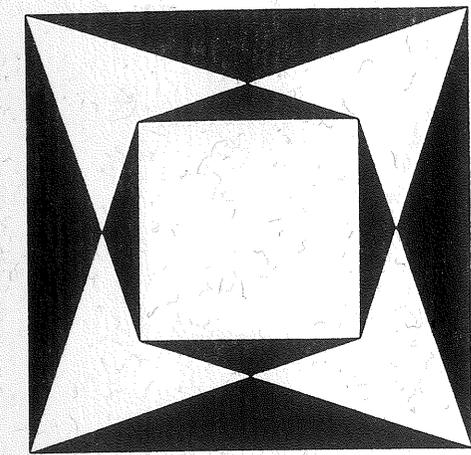


Dr D.W. Lewis
Department of Mathematics
University College
Belfield
DUBLIN 4

IRISH MATHEMATICAL SOCIETY



BULLETIN

NUMBER 27 DECEMBER 1991
ISSN 0791-5578

IRISH MATHEMATICAL SOCIETY
BULLETIN

EDITOR: Dr J. Ward
ASSOCIATE EDITOR: Dr R. Dark
BOOK REVIEW EDITOR: Dr M. Tuite
PROBLEM PAGE EDITOR: Dr P. Rippon
PRODUCTION MANAGER: Dr M. Ó Searcóid

The aim of the Bulletin is to inform Society members about the activities of the Society and about items of general mathematical interest. It appears twice each year, in March and December. The Bulletin is supplied free of charge to members; it is sent abroad by surface mail. Libraries may subscribe to the Bulletin for IR£20.00 per annum.

The Bulletin seeks articles of mathematical interest written in an expository style. All areas of mathematics are welcome, pure and applied, old and new. The Bulletin is typeset using TeX. Authors are invited to submit their articles in the form of TeX input files if possible, in order to ensure speedier processing.

Correspondence concerning the Bulletin should be addressed to:

Irish Mathematical Society Bulletin,
Department of Mathematics,
University College,
Galway,
Ireland.

Correspondence concerning the Problem Page should be sent directly to the Problem Page Editor at the following address:

Faculty of Mathematics,
Open University,
Milton Keynes, MK7 6AA,
UK.

The Irish Mathematical Society acknowledges the assistance of EOLAS, The Irish Science and Technology Agency, in the production of the Bulletin.

CONTENTS

IMS Officers and Local Representatives	ii
Notes on Applying for IMS Membership	iii
Minutes of IMS meeting 28.3.91	1
September 1991 meeting	Graham Ellis 3
Thesis Abstracts	
Minimality via Order and Topology .. Aisling E. McCluskey	4
Dimensions of commutative matrix algebras . Susan Lazarus	7
Research Announcement	
Preduals of Spaces of Holomorphic Functions	C. Boyd 9
Directions for Contributors	
Proposed Solution to a Problem of Minimal	
Direction in Typesetting	Mícheál Ó Searcóid 13
Articles	
100 Years of Dixon's Identity	James Ward 46
Some Groups of Exponent p	J. D. Reid 55
Book Review	
Mathematics and its History	James Ward 63

THE IRISH MATHEMATICAL SOCIETY

Officers and Committee Members

President	Dr R. Timoney	School of Mathematics Trinity College Dublin
Vice-President	Dr B. Goldsmith	Department of Mathematics Kevin Street College Dublin
Secretary	Dr G. Ellis	Department of Mathematics University College Galway
Treasurer	Dr D. A. Tipple	Department of Mathematics University College Dublin

Committee Members:

Dr G. Enright, Dr F. Gaines, Prof. F. Holland, Dr B. McCann,
Prof. A. O'Farrell, Dr M. Ó Searcóid, Prof. D. Simms, Dr J. Ward,
Dr R. Watson.

Local Representatives

Cork	RTC	Mr D. Flannery	
	UCC	Dr M. Stynes	
Dublin	DIAS	Prof. J. Lewis	
	Kevin St.	Dr B. Goldsmith	
	DCU	Dr M. Clancy	
	St. Patrick's	Dr J. Cosgrave	
	TCD	Dr R. Timoney	
Dundalk	UCD	Dr F. Gaines	
	RTC	Dr E. O'Riordan	
	Galway	UCG	Dr R. Ryan
	Limerick	MICE	Dr G. Enright
	UL	Dr R. Critchley	
Maynooth	Thomond	Mr J. Leahy	
		Prof. A. G. O'Farrell	
	Waterford	RTC	Mr T. Power
Belfast	QUB	Dr D. W. Armitage	

NOTES ON APPLYING FOR I.M.S. MEMBERSHIP

1. The Irish Mathematical Society has reciprocity agreements with the American Mathematical Society and the Irish Mathematics Teachers Association.
2. The current subscription fees are given below.

Institutional member	IR£50.00
Ordinary member	IR£10.00
Student member	IR£4.00
I.M.T.A. reciprocity member	IR£5.00

The subscription fees listed above should be paid in Irish pounds (pint) by means of a cheque drawn on a bank in the Irish Republic, a Eurocheque, or an international money-order.

3. The subscription fee for ordinary membership can also be paid in a currency other than Irish pounds using a cheque drawn on a foreign bank according to the following schedule:

If paid in United States currency then the subscription fee is US\$18.00.

If paid in sterling then the subscription fee is £10.00 stg.

If paid in any other currency then the subscription fee is the amount in that currency equivalent to US\$18.00.

The amounts given in the table above have been set for the current year to allow for bank charges and possible changes in exchange rates.

4. Any member with a bank account in the Irish Republic may pay his or her subscription by a bank standing order using the form supplied by the Society.
5. The subscription fee for reciprocity membership by members of the American Mathematical Society is US\$10.00.

6. Subscriptions normally fall due on 1 February each year.
7. Cheques should be made payable to the Irish Mathematical Society. If a Eurocheque is used then the card number should be written on the back of the cheque.
8. Any application for membership must be presented to the Committee of the I.M.S. before it can be accepted. This Committee meets twice each year.
9. Please send the completed application form with one year's subscription fee to

The Treasurer, I.M.S.
Department of Mathematics
University College
Dublin
Ireland

Minutes of Meetings of the Irish Mathematical Society

Ordinary Meeting

28 March 1991

The Irish Mathematical Society held an Ordinary Meeting at 12.45 pm on Thursday 28th March 1991 in the DIAS.

Eleven members were present. The President, R. Timoney was in the chair. The Secretary sent his apologies, and M. Newell took minutes.

1. The **minutes** of the meeting of 21st December 1990 were approved and signed.
2. **Matters arising:** The poster for the 1991 Galway September Meeting was circulated.
Waterford RTC agreed to host the 1992 September Meeting. Brendan McCann was nominated as the contact person.
The Bulletin is to appear shortly.
The Committee agreed some minor changes in the rules of the Society and will circulate these.
3. There was no **correspondence**.
4. **European Mathematical Society:** Brendan Goldsmith outlined the nature of the EMS and encouraged members to become individual members at a fee of about £11. Council recommends accepting this offer. He gave details of the Eurocongress in July 1992; further information has been circulated electronically to Math. Dept. list.
5. **Points for Leaving Certificate Maths:** The President reported on press articles and ministerial comments on the issue. He informed the meeting of a formal response which he had published in the Irish Times and Evening Press. In a long discussion, in which almost all members present participated, it was suggested that the IMS should adopt a policy of encouraging pupils to take Honours Mathematics courses.

It was generally felt that the need for bonus points should be explained in the context of the centrality of the subject to science. It was suggested that the content of the course should be changed, and also that the subject should be recognized as a double subject (two papers).

6. **AOB:** D. O'Donovan circulated copies of the Survey of Trinity College graduates. The Committee recommends other institutions carry out similar surveys.

Martin Newell,
University College,
Galway.

THE IMS SEPTEMBER 1991 MEETING

The Irish Mathematical Society held its annual mathematical meeting in University College Galway, on Thursday 5th and Friday 6th September 1991. Forty two people registered for the meeting.

The opening lecture was given by Daniel Mauldin (University of North Texas) on "Deterministic and random fractals". After lunch, Paddy Barry (UCC) spoke on "Some aspects of projective and Euclidean geometry". He was followed by Pauline Mellon (Maynooth), who spoke on "Dual symmetric manifolds". The final lecture on the Thursday was given by Jean-Louis Loday (Institut de Recherche Mathématique Avancée, Strasbourg) on "Non-abelian homology of Lie algebras". A barbecue was held in the evening.

Rüdiger Göbel (Universität Essen) began the Friday session with a lecture on "Vector spaces with distinguished subspaces". This was followed by two parallel splinter sessions. The following gave splinter talks: Paul Barry (UCD), Seán Dineen (UCD), Fergus Gaines (UCD), Brendan McCann (Waterford), Richard Timoney (TCD), Michael Tuite (UCG). After lunch Seán Tobin (UCG) spoke on "Burnside revisited". The meeting ended with a lecture by Charles Nash (Maynooth) on "Topological field theories".

A book exhibition was open throughout the meeting. Several publishers contributed to this, and offered discounts of up to 20%.

Graham Ellis,
University College,
Galway.

Abstract of Doctoral Thesis

MINIMALITY VIA ORDER AND TOPOLOGY

Aisling E. M^cCluskey

This thesis was prepared under the supervision of Dr S. D. M^cCartan in Queen's University Belfast. The external examiner was Professor Stephen Watson (York University, Toronto), and the degree of Ph.D. was awarded in December 1990.

The family of all topologies definable for an infinite set X is a complete, atomic and complemented lattice (under inclusion) and is often denoted by $LT(X)$. Given a topological invariant P , a member \mathcal{T} of $LT(X)$ is said to be minimal (maximal) P if and only if \mathcal{T} possesses property P but no weaker (stronger) member of $LT(X)$ possesses property P .

The underlying theme of this work, then, is an investigation of minimality with respect to certain topological properties, including that of sobriety. (A space has the latter property precisely when every non-empty closed irreducible subset is a unique point-closure.) Motivation for such an investigation is provided by realizing that it is in seeking to identify those members of $LT(X)$ which minimally satisfy an invariant property that we are, in a very real sense, examining the topological essence of the invariant. In the recent past, questions of this nature have been considered by Andima and Thron [1], Larson [4], M^cCartan [5] and Johnston and M^cCartan [2,3].

The thesis comprises three main sections, the first two of which are devoted to a discussion of minimality using a purely topological approach. In particular, we consider sobriety, an invariant which has seen striking application in such seemingly diverse areas as the theory of continuous lattices and theoretical computer science (notably domain theory). Indeed, recent developments in these fields would justify the claim that life without



Hausdorff is not only possible but that it is imperative. Research efforts in domain theory have been significantly advanced by the recognition of the limitations imposed by a traditional insistence on Hausdorff.

The final section represents a shift in focus where we give an order-theoretic interpretation of the previously established minimality structures. The order referred to here is the natural partial order induced on X by any T_0 -member of $LT(X)$, thus:

$$x \leq y \text{ if and only if } x \in \overline{\{y\}} \quad x, y \in X.$$

Given a topological space then, we may invoke the inherent partial order and rewrite point-closures and point kernels order-theoretically as:

$$\begin{aligned} \overline{\{y\}} &= \{z \in X : z \leq y\} \\ \widehat{\{y\}} &= \{z \in X : z \geq y\}, \end{aligned} \quad y \in X.$$

Thus, we have at our disposal a partial order which may be readily exploited and indeed lends a welcome visual aspect to the discussion. In particular, the structure of invariants expressed solely in terms of point-closures is reflected in the nature of the induced partial order. Thus they may be interpreted as properties of partial orders, which offers new insight into the problem of minimality. This is the tack taken by Andima and Thron in [1] where they consider certain 'order-induced' topological properties; that is, a topological property P which has an associated order property K such that a topology has property P if and only if its induced order has property K . Thus in searching for minimality with respect to such a property, we are now concerned with establishing the appropriate K -order structure.

Of course, many invariants are not of this type but nevertheless, an awareness of the effect of the invariant on the behaviour of the underlying partial order provides valuable insight into the methods employed to establish minimality.

An important aspect of this work is the development of certain techniques to solve many of the minimality problems under

consideration. We illustrate their potential in characterizing and, where possible, identifying certain minimal structures. Further, while these methods are introduced in a purely topological setting, we show that they have a strong order-theoretic appeal. Their topological significance has a direct order-theoretic translation when we regard the space as a partially-ordered set.

References

- [1] S. J. Andima and W. J. Thron, *Order-induced topological properties*, Pacific J. Math. (2) 75 (1978).
- [2] B. Johnston and S. D. M^CCartan, *Minimal T_F -spaces and minimal T_{FF} -spaces*, Proc. R. Ir. Acad. 80A (1980), 93–96.
- [3] B. Johnston and S. D. M^CCartan, *Minimal T_{YS} -spaces and minimal T_{DD} -spaces*, Proc. R. Ir. Acad. 88A (1988), 23–28.
- [4] R. E. Larson, *Minimal T_0 -spaces and minimal T_D -spaces*, Pacific J. Math. 31 (1969), 451–458.
- [5] S. D. M^CCartan, *Minimal T_{ES} -spaces and minimal T_{EF} -spaces*, Proc. R. Ir. Acad. 79A (1979), 11–13.

Aisling E. M^CCluskey,
University College,
Galway.

Abstract of Doctoral Thesis

DIMENSIONS OF COMMUTATIVE MATRIX ALGEBRAS

Susan Lazarus

This thesis was prepared under the direction of Professor T. Laffey in University College Dublin. The external examiner was Professor José Dias de Silva and the degree of Ph.D. was awarded by the NUI in Autumn 1991.

Let F be a field and let $M_n(F)$ be the algebra of $n \times n$ matrices over F . Let $A, B \in M_n(F)$ with $AB = BA$ and let \mathcal{A} be the algebra generated by A and B over F . A theorem of Gerstenhaber [Ann. Math. 73: 324–348 (1961)] states that the dimension of \mathcal{A} is at most n . Gerstenhaber's proof uses the methods of algebraic geometry. In Chapter I of this thesis, we obtain a purely matrix-theoretic proof of the result, constructing in the process a basis for the algebra, \mathcal{A} . We also examine when equality occurs. The case where F is algebraically closed and \mathcal{A} is indecomposable (under similarity) holds the key to the general situation. In this case, we obtain a Cayley–Hamilton–like theorem expressing B^k as a polynomial in I, B, \dots, B^{k-1} with coefficients in $F[A]$, where k denotes the number of blocks in the Jordan form of A . If all Jordan blocks of A have the same size, we say \mathcal{A} is homogeneous. In this case we obtain a nonderogatory–like condition on B which is equivalent to $\dim_F \mathcal{A} = n$. We also show that in this case, $\dim_F \mathcal{A} = n$ is equivalent to the maximality of \mathcal{A} as a commutative subalgebra of $M_n(F)$.

In Chapter II we examine the dimensions of three-generated commutative subalgebras of $M_n(F)$. Let A, B and $C \in M_n(F)$ be pairwise commutative, and let \mathcal{A} be the algebra generated by A, B and C over F . It is an open question whether or not the dimension of \mathcal{A} is bounded above by n . Again, the case where

F is algebraically closed and \mathcal{A} is indecomposable holds the key concepts. If A , say, has r Jordan blocks, with the biggest Jordan block of size $k \times k$, then it is shown that generally $\dim_F \mathcal{A} \leq \{nk, kr(r+1)/2\}$. In the homogeneous case, it is shown that $\dim_F \mathcal{A} \leq n^{3/2}$, and if \mathcal{A} has fewer than four Jordan blocks, then $\dim_F \mathcal{A} \leq n$. Further if the exponent of the algebra \mathcal{A} is also k (i.e. $\mathcal{A}^k = 0$), then it is shown that for $n < 30$, $\dim_F \mathcal{A} \leq n$. In case \mathcal{A} is homogeneous, then each matrix in \mathcal{A} can be considered as an element of $M_r(F[J])$ (where $A = J \oplus \dots \oplus J$, r blocks of $J = J_k$, the $k \times k$ Jordan block with associated eigenvalue zero). It is shown that if B is a Wasow matrix over the local commutative ring $F[J]$, i.e., B is similar over $F[J]$ to a matrix in rational canonical form, then again in this case the dimension of \mathcal{A} cannot exceed n .

Let \mathcal{A} be a commutative subalgebra of $M_n(F)$, and say the centralizer of \mathcal{A} , $\mathcal{C}(\mathcal{A})$, is contained in \mathcal{A} . Then \mathcal{A} is said to be a maximal commutative subalgebra of $M_n(F)$. We define the exponent of \mathcal{A} to be the smallest positive integer k such that $x_1 \dots x_k = 0$ for all x_1, \dots, x_k in the radical of \mathcal{A} . In Chapter III we study the dimensions of maximal commutative subalgebras of $M_n(F)$. A classical result of Schur states that $\dim_F \mathcal{A} \leq [1 + n^2/4]$, where $[\]$ denotes the greatest integer function. Courter [Duke Math. J. 32:225-232 (1965)] proved if \mathcal{A} has exponent two then $\dim_F \mathcal{A} \geq n$. Laffey [Linear Alg. Appl. 71:199-212 (1985)] showed that generally $\dim_F \mathcal{A} \leq (2n)^{2/3} - 1$, and if \mathcal{A} has exponent three then the best possible lower bound is $[3n^{2/3} - 4]$. We create a sequence of maximal commutative subalgebras \mathcal{A}_n , each with exponent four, with $\dim_F \mathcal{A}_n$ of the order of $n^{2/3} - n^{1/3}$, in the limit. On the other hand, if the exponent of \mathcal{A} is greater than or equal to $n-3$, and the characteristic of F does not divide $n!$, then we show that $\dim_F \mathcal{A}$ is either n , $n+1$ or $n+2$.

Susan Lazarus,
Dublin Institute of Technology,
Kevin Street,
Dublin 8.

Research Announcement

PREDUALS OF SPACES OF HOLOMORPHIC FUNCTIONS

Christopher Boyd

For U an open subset of a locally convex space E we denote by $\mathcal{H}(U)$ the space of \mathbb{C} -valued holomorphic functions on U . In infinite dimensional holomorphy we consider three natural topologies on $\mathcal{H}(U)$. τ_o is the compact-open topology of convergence on compact subset of U . We say a semi-norm p is ported by the compact subset K of U if for each open set V , $K \subset V \subset U$, we can find $c(V) > 0$ such that $p(f) \leq c(V) \|f\|_V$ for every f in $\mathcal{H}(U)$. τ_ω is the topology generated by all semi-norms ported by compact subsets of U . Finally say that a semi-norm p is τ_δ continuous if for each countable increasing open cover $\{U_n\}_n$ of U there is $C > 0$ and $n_o \in \mathbb{N}$ such that $p(f) \leq C \|f\|_{U_{n_o}}$ for every $f \in \mathcal{H}(U)$. τ_δ is the topology on $\mathcal{H}(U)$ generated by all τ_δ continuous semi-norms. We always have

$$\tau_o \leq \tau_\omega \leq \tau_\delta$$

on $\mathcal{H}(U)$. $P^n(E)$ denotes the space of n -homogeneous polynomials on E . We note that τ_ω and τ_δ agree on $P^n(E)$ for every integer n . For K a compact subset of E we let $\mathcal{H}(K)$ denote the space of holomorphic germs on K . The τ_o (resp. τ_ω) topology on $\mathcal{H}(K)$ is defined by $(\mathcal{H}(K), \tau_o) = \text{ind}_{K \subset V} (\mathcal{H}(V), \tau_o)$ (resp. $(\mathcal{H}(K), \tau_\omega) = \text{ind}_{K \subset V} (\mathcal{H}(V), \tau_\omega)$).

Given a locally convex space E we let $E'_i = \text{ind}_V E'_V$, where the inductive limit is taken over all neighbourhoods V of 0 in E , and let E'_b denote the dual of E equipped with the topology of uniform convergence on bounded subsets of E .

In [3] Mujica and Nachbin shows there is a complete locally convex space $G(U)$ with the property that $G(U)'_i = (\mathcal{H}(U), \tau_\delta)$.

By construction

$$(\mathcal{H}(U), \tau_o)'_b \subseteq G(U) \subseteq (\mathcal{H}(U), \tau_\delta)'_b.$$

We consider a special case, when U is a balanced open subset of a Fréchet (complete metrizable) space E , and show how the topological properties of $G(U)$ are related to the topological properties of E . In this situation we note that $G(U)$ is neither metrizable nor dual metrizable except in the trivial case when E is finite dimensional.

Reflexivity

Firstly we consider the question: When is $G(U) = (\mathcal{H}(U), \tau_\delta)'_b$? The following theorem characterizes U with this property.

Theorem 1. For a Fréchet space E the following are equivalent:

- (i) $(P^n E, \tau_o)' = (P^n E, \tau_\omega)'$ for every integer n .
- (ii) $(\mathcal{H}(U), \tau_o)' = (\mathcal{H}(U), \tau_\omega)'$ (resp. $(\mathcal{H}(K), \tau_o)' = (\mathcal{H}(K), \tau_\omega)'$) for one and hence every balanced open (resp. compact) subset U (resp. K) of E .
- (iii) $(P^n E, \tau_\omega)$ is reflexive for every integer n .
- (iv) $(\mathcal{H}(U), \tau_\omega)$ is semi-reflexive for one and hence every balanced open subset U of E .
- (v) $(\mathcal{H}(U), \tau_\delta)$ (resp. $(\mathcal{H}(K), \tau_\omega)$) is reflexive for one and hence every balanced open (resp. compact) subset U (resp. K) of E .
- (vi) $G(U) = (\mathcal{H}(U), \tau_\delta)'_b$ for one and hence every balanced open subset U of E .
- (vii) $(\mathcal{H}(U), \tau_\omega)'_b \subseteq G(U)$ for one and hence every balanced subset U of E .

Alencar, Aron and Dineen showed in [1] that $(\mathcal{H}(U), \tau_\delta)$ is reflexive for every balanced open subset of Tsirelson's space. Therefore there are Banach spaces with unconditional basis for which $G(U) = (\mathcal{H}(U), \tau_\delta)'_b$. The above Theorem and a result of Aron (see [4]) also allow us to show that $G(U) \neq (\mathcal{H}(U), \tau_\delta)'_b$ when U is a balanced open subset for any ℓ_p space.

Distinguishedness

Although $G(U)$ may not be equal to $(\mathcal{H}(U), \tau_\delta)'_b$ it still may be possible that the inductive dual of $G(U)$, $G(U)'_i$ is equal to the strong dual of $G(U)$, $G(U)'_b$. When this occurs we have $(\mathcal{H}(U), \tau_\delta) = G(U)'_b$. To see when it does happen we need the notion of distinguishedness.

A locally convex space E is said to be distinguished if every $\sigma(E'', E')$ -bounded set of E'' is contained in the $\sigma(E'', E')$ -closure of some bounded set of E . If E is a Fréchet space then E is distinguished if and only if $E'_i = E'_b$. When U is a balanced open subset of a Fréchet space, $G(U)$ satisfies the following analogous theorem.

Theorem 2. Let U be a balanced open subset of a Fréchet space E , then $G(U)$ is distinguished if and only if $G(U)'_i = G(U)'_b$.

It can be shown that $G(U)$ is distinguished for every balanced open subset of a Banach space with an unconditional basis. In particular this will mean that $G(U)'_b = (\mathcal{H}(U), \tau_\delta)$ for every balanced open subset of ℓ_p , $0 \leq p < \infty$.

Quasinormability

Grothendieck [2] introduced the notion of quasinormability as a property which was stable under a large number of topological vector space operations (e.g. formation of biduals, projective tensor products, etc).

A locally convex space E is said to be quasinormable if for each absolutely convex neighbourhood U of 0 in E there is an absolutely convex neighbourhood of 0, V , such that given $\alpha > 0$ there is a bounded subset B_α of E with

$$V \subset B_\alpha + \alpha U.$$

Every normed space is quasinormable, while a Fréchet-Montel space is quasinormable if and only if it is Schwartz. We have the following theorem concerning the quasinormability of $G(U)$.

Theorem 2. *Let U a balanced open subset of a Fréchet space E , then $G(U)$ is quasinormable if and only if E is quasinormable.*

These results are part of the authors doctoral thesis under Prof. S. Dineen, which is due to be submitted to the NUI in 1992.

References

- [1] R. Alencar, R. Aron and S. Dineen, *A Reflexive Space of Holomorphic Functions in Infinitely Many Variables*, Proc. Amer. Math. Soc. 90 (1984), 407–411.
- [2] A. Grothendieck, *Sur les espaces (F) et (DF)* , Summa Bras. Math. 3(6) (1954), 57–122.
- [3] J. Mujica and L. Nachbin, *Linearisation of Holomorphic Mappings on Locally Convex Spaces* (1992) (Preprint).
- [4] R. Ryan, *Applications of Topological Tensor Products to Infinite Dimensional Holomorphy*. Trinity College Dublin, 1980 (Doctoral thesis).

Christopher Boyd,
Department of Mathematics,
University College Dublin,
Belfield,
Dublin 4.

PROPOSED SOLUTION TO A PROBLEM OF MINIMAL DIRECTION IN TYPESETTING

Notes for contributors

Mícheál Ó Searcóid

Abstract: This is a first attempt to present to contributors of the Bulletin a plain TEX package which will allow them to do most of their own editing and to present papers in a form more or less ready for publication. The article also describes how to use *MISTRESS*, a system for typesetting references in many journals with different styles.

Introduction

The Bulletin has been beset by production problems over the last couple of years. Rex Dark and I, as acting editor and production manager, decided that we should make a concerted effort to tackle these problems in a way which would both enable us to get the Bulletin back on schedule by early 1993 and make it easier for future editors to get issues out on time. In this article, I should like firstly to make some observations and suggestions to the mutual benefit, I hope, both of the production team of the Bulletin and of its contributors. Secondly, I will describe the solution to the problems which arose from those observations.

Submissions to the Bulletin

It was decided at a recent meeting of the IMS that contributors should be encouraged to submit articles in TEX if they have the facilities to do so. Since our typesetting is done in TEX , it is reasonable to suppose that TEX -written papers will be processed more quickly than those that are not so written; indeed some journals

now quantify the extra time that non- \TeX authors are likely to have to wait to see their work in print. It appears, however, that this is no longer the most important consideration for speedy production of the Bulletin.

Judging by the material in the 1991 issues and by that which remains in the editor's file, it is evident that most of our contributions are now presented in \TeX . Of all this material however, only one review looked as it came from the author even remotely like the finished product in print. We have plain \TeX , \LaTeX and \AMS-TeX , each used with a multitude of different styles, magnifications and fonts, and each in a format pleasing, no doubt, to the eye of the author. Nonetheless, an editor must impose some degree of uniformity on the works he is going to print. Here then is the nub of the problem: most of us are now typesetting our own material, but the IMS has not yet got round to telling us, even in broad terms, how it wants it to be done.

What guidelines should there be for the typesetter? There is one obvious physical limitation: a paper typeset in non-magnified fonts on A4 paper can hardly be squeezed onto A5 and remain easily readable. There are other limitations demanded by good taste: that headlines, numbering and title fonts be consistent; that page dimensions be consistent; and that references be made in a standard journal style. Information regarding these and similar details, if made available to the contributor, would enable him to decide how his long equations would be broken, where his pagebreaks would occur, and generally what the final version of his paper would look like in print. In short it would increase and not decrease his freedom and independence.

Moreover, since contributors have, for the most part, already acceded to the express wish of the Society that articles be submitted in \TeX , there is every reason to suppose that, if presented with a basic format file and instructions on how to use it, they would be willing, and indeed find it to their own advantage, to employ such a file in preparing their documents. For better or worse, we mathematicians are now our own typesetters, and we want to be able to present copy which needs little or no editing.

It seems, therefore, that any long term solution to our prob-

lems of production must include a freely available format file. If such a file is to gain general acceptance, however, it must be in some sense minimal; it must at the very least not make the task of writing for the Bulletin more difficult than it is at present. I have therefore set about producing a file based on these considerations and on data extrapolated from earlier issues of the Bulletin, and I have tried to make it as easy as possible to use. The result is `imsform.tex` which has been tested on two issues and has received editorial approval. It is described fully in this article. Instructions on how to obtain it electronically are contained in the **Final Notes** section of the article.

The most complex part of the operation, that of ensuring accuracy and consistency in the typesetting of reference lists, has been approached from a much more general point of view, and the resulting macro file `mistress.tex` may be used when writing papers for journals with many different styles.

Title heading

An author can never know where pagebreaks are going to occur in a document unless he can typeset the heading of the document exactly as it will appear in print. A number of macros are available in `imsform.tex` which give \TeX the header information. Before listing them, we show the beginning of the input file for this article, by way of example:

```
\input mistress
\input imsform
%
\imsauthor
M\'\i che\'al \'O'Searc\'oid

%
\imstitle
PROPOSED SOLUTION
TO A PROBLEM OF MINIMAL DIRECTION
IN TYPESETTING

%
```



```
\imssubtitle
Notes for contributors
```

```
%
\imsshorttitle{Direction in Typesetting}%
```

```
%
\imsaddress
M\ 'i che\ 'al \ 'O~Searc\ 'oid,
Roinn na Matamaitice,
Col\ 'aiste na hOllscoile,
Baile \ 'Atha Cliath.
```

```
%
\imsabstract{This is a first attempt ... styles.}%
```

```
%
\imsstart
```

Here is the information needed for using the header macros:

1. At the head of the document the file `imsform.tex` should be input. If references are to be included, then the file `mis-tress.tex` should also be input.
2. Each of the control sequences `\imstitle`, `\imsauthor`, `\imssubtitle`, `\imsaddress`, together with two others not used in this article, `\imsovertitle` and `\imsrevinfo`, must, if used, appear alone on a line, should be followed by lines of text which are to appear in the output on distinct lines and should be followed by a blank line. Lines of output will be centred automatically, with the exception of those that go with `\imsaddress`.
3. In expanding any of the macros mentioned above, `TEX` will use the end of line marker as a parameter terminator. A comment marker `%` on any of these lines of text would prevent `TEX` from seeing such a marker. It follows that comments should not be made between the initial control sequence and the terminating blank line when any of these macros is being used.
4. All lines of text associated with these macros should normally be written in lower case, except for those belonging



with `\imstitle`.

5. In general a contributor should allow `\imsform` to decide which fonts are going to be used in the heading.
6. Sometimes an extra heading will be necessary above the title. It is for typesetting such a heading that the control sequence `\imsovertitle` is intended. Note its use for the heading `Book Review` in this issue of the Bulletin. Since such overtitles are typeset in the mandatory white space which always appears above the title of an article, they do not take up any extra space. Their insertion or removal by the editor will not therefore affect page breaks.
7. The control sequence `\imsrevinfo` is intended for book reviews. The information given here might include the name of the publisher, the place of publication, the price, the number of pages, or other relevant data in accordance with the wishes of the editor. The reviewer has the opportunity to determine how this information should be split into lines, bearing in mind that these lines will be centred on the page.
8. In the case of multiple authorship, the names of all authors should appear in a single use of the macro `\imsauthor`. The various names will be distributed onto separate lines exactly as is done in the input file.
9. Different addresses should be entered separately. So, if two or more authors have separate addresses, the macro `\imsaddress` should be used once for each address. The names of the authors should be included in the address. A roman typeface will be used automatically for the addresses, and they will be typeset at the end of the paper. Actually it is the incidence of multiple authorship which makes it desirable to have a macro of this sort, because in that case one wants, if possible, to have some addresses left justified and others right justified. The macro `\imsaddress` decides whether or not the lines of the addresses are too long for this and, if not, arranges pairs of addresses to hang from the same horizontal line.
10. The other header macros are `\imsreviewer`, `\imsabstract`, `\imsshorttitle` and `\imsinfo`. Each of these should be fol-



- lowed by a single parameter enclosed in braces.
11. An example of the use of `\imsabstract` is given above. The use of `\imsinfo` is identical. The difference in output between these two macros is that the former will typeset the word **Abstract** before the text whereas the latter will not. The macro `\imsinfo` has been used in issue 26 and in this issue in the abstracts of doctoral theses. Note that abstracts appear in nine point type. The reason for this is simply to allow them to match any references, which for technical reasons appear in the smaller type.
 12. The control sequence `\imsreviewer` should be used in book reviews and should be followed by the name of the reviewer flanked by braces. In the output, the reviewer's name will be automatically preceded by the words 'Reviewed by'.
 13. The control sequence `\imsshorttitle` should be followed by the headline which is to appear at the top of pages. This must not be wider than about three quarters of the length of a line of text in the Bulletin. The short title will appear in a slanted typeface at the head of every odd-numbered page which is not also the first page of the article. Even-numbered pages will show the name and issue of the Bulletin. In the proof copy this will read *IMS Bulletin 00, 1978*, but that will be corrected by the editorial staff at the time of publication.
 14. When all header macros have been used, the control sequence `\imsstart` should be written and the body of the article begun. This macro typesets the header with all the desired spacings and leaves \TeX in horizontal mode, suppressing indentation of the subsequent text.
 15. No font changes or vertical skips should be introduced before, during or after use of the heading macros; those required by the editor have been built into the macro `\imsstart`.

General layout

Let us accept as a premise that most mathematicians are reasonably competent typesetters. This may not be entirely true, but it is not unlikely that many are at least as competent as the layout



editor. It seems best, therefore, at this stage, to present only a few general guidelines as to how a paper should be typeset.

1. Authors are reminded that indentation of paragraphs in English language typesetting never applies to the first paragraph of a book or paper. Plain \TeX does not automatically suppress that indentation, but the IMS macro `\imsstart` always leads into a new paragraph and always suppresses indentation of the following text.
2. Next, if the paper falls naturally into sections, as this article does, then the author is asked to use the plain \TeX macro `\beginsection` described on page 340 of the \TeX book. Note that the section heading following the control sequence `\beginsection` must always be followed by a blank line; moreover indentation of the succeeding paragraph is always inhibited. The heading is automatically set in bold type. This section, for example, was begun with

```
\beginsection General layout
```

Let us accept...

Knuth arranges for \TeX to find out if there is very much of the page left before beginning to typeset a new section; if there is not, \TeX will move on to a new page. I have changed Knuth's settings, so that a new page will be started only if there is very little room left on the present page.

3. For theorems, lemmas etc., contributors are asked to make use of the plain \TeX macro `\proclaim` which is described on page 340 of the \TeX book. `imsform` provides macros `\imsproof` and `\imsendproof`. The former does a small vertical skip, starts a new paragraph without indentation, writes *Proof:* and leaves a `\quad` of space before the text which follows. The latter places a small square box at the right end of the current line (or at the right end of the next line if there is no room there) and creates a medium sized vertical skip.
4. For the sets of natural numbers, rational numbers, real numbers, complex numbers and so on, authors are asked to use

- bold face upper case letters (N, Q, R, C etc.).
5. The package `imsform.tex` contains a macro `\ninepoint` which makes all the changes necessary for typesetting both text and mathematics in the smaller point size. The meanings of control sequences such as `\rm`, `\tt`, `\sl`, `\it` etc. are all altered, as are all the spaces used in typesetting. The `\ninepoint` macro should be used sparingly for displayed material. `imsform.tex` contains also a `\tenpoint` macro.
 6. Articles should end with `\bye`, not with `\end`. `imsform.tex` has redefined plain TeX's macro `\bye` so that it performs as usual when typesetting a single file, but behaves like `\endinput` when the editor typesets a complete issue.
 7. At the end of the paper, before saying `\bye`, the contributor should type `\imsclose`. This macro will typeset the addresses previously given; it will also activate the *MISTRESS* control sequence `\References` if `mistress.tex` has been input. All spacing is automatic.
 8. Authors should be careful about the use of private macros. Since a whole issue of the Bulletin will be typeset eventually from a single input file, macros defined in one article may affect the typesetting of another. Private macros must not interfere with public ones. In particular, authors should not use control sequences which override similar ones defined in plain TeX. Since IMS macros normally begin with `\ims`, it is easy to avoid confusion with them. Also, since *MISTRESS* macros begin with upper case letters, it is best to reserve such control sequences for references, as described later in this article. It should also be noted that certain combinations of characters signal ligatures to the CM fonts, so should be avoided; for example, `\i` should not be used as a macro name.
 9. When the editor processes the main file for a complete issue of the Bulletin, the list of contents will be automatically typeset. An author can specify a contents entry by using `\imscontadd` somewhere in his file. The entry for this article was

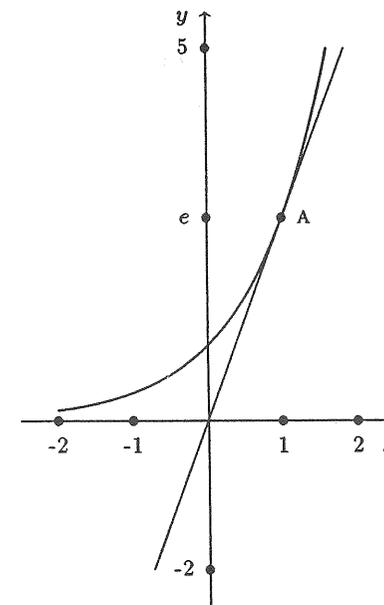
```
\imscontadd{Proposed Solution to a Problem
of Minimal Direction in Typesetting&M'\i che\al
\ '0~Searc\ 'oid&\imspageno}%
```

The title comes first; if it is too long for a line, `imsform` will split it. It will not do the same for the author or authors, which come next after a single ampersand. Where there is more than one author, it might be desirable to typeset the names on separate lines; this can be done by enclosing the authors' names in braces and separating them by ampersands, e.g. `{J.~Smith&W.~Berry}`. The names should be followed by an ampersand and the control sequence `\imspageno`.

Typesetting diagrams

A few points should be remembered when introducing diagrams into a paper:

1. The Bulletin is published in a rather small format which means that diagrams should in general be somewhat smaller than they might be in some other journals.
2. Diagrams usually consist of big chunks of typescript. It may often be necessary to juggle around a little with the raw text of a document so that each diagram is typeset not only at a relevant point in the article but also at a suitable point on a page to avoid vertical overflow.
3. If an author has used THE GRAFFAK to create diagrams for a Bulletin article, it may be well to remember that although diagrams are typeset



This diagram was typeset using the command `\Diagram1`; the text you are reading followed, in braces. The exact position of the whole picture was arranged by inspection.



simply by writing `\diagram1`, `\diagram2` etc., there is provision in THE GRAFFAK for the author to vary the amount of space automatically inserted above and below diagrams and also that space between the diagram and any legend which might accompany it.

4. If diagrams created by THE GRAFFAK are to be typeset, the author should submit, with the \TeX source file for his article, the METAFONT and \TeX files written by THE GRAFFAK which form the source files for his diagrams.

Typesetting references

It is characteristic of the pure mathematician to seek general solutions to particular problems. It was not my job to decide on a format for references in the Bulletin; rather I have followed what, in general, the editors have been doing, not without some loss of consistency, in recent issues. It was somewhat harder to decide how to make it easy (the emphasis must be on ease) for an author to conform to such a style. Moreover references are things which may be used several times in several different publications, whose editors may and do demand that their own particular styles be adhered to. Styles differ considerably, yet we should like to be able to lift a reference directly from one paper for use in another. In any case, it is probably true that more typing errors are made and overlooked in reference lists than anywhere else in a paper; any automated system to ensure correct syntax and punctuation is surely worth having. Yet plain \TeX makes no particular provision for typesetting references, nor was there readily available any macro package to assist us in doing it. So the general solution which arose from the particular problem of typesetting reference lists for the IMS Bulletin is *MISTRESS: Macros of an Intelligent System for the Typesetting of References in accordance with External Specifications of Syntax*.

Introducing *MISTRESS*

Where references are to appear at the end of a document, the invocation `\input mistress` should appear somewhere near the



beginning of the file. The references themselves should be contained in a separate file whose name is the same as the name of the \TeX source file of the document, but with `.ref` as extension.

The references should be written in a standard format explained below; references written in this format can subsequently be typeset for another journal provided you have a syntax file for that journal (fortunately these are not too difficult to write, as will be explained later).

In fact *MISTRESS* is empowered to instruct \TeX to write the reference file for any particular document, provided the references needed are already contained in other master reference files. The format of any particular reference in a master file and its copy in the specific file for a document are identical. For the moment, therefore, we shall suppose that the `.ref` file for our document has been written. In the next section, we shall explain how to tell \TeX to write it.

The reference file for the present article is as follows:

```
\Syntax {original}
\Newref CAR87a
Carroll*
\Names M. M. Carroll
\Status in press
\Article Exact solutions in compressible isotropic %
elasticity
\Editor M. M. Carroll
\Inbook Nonlinear effects in solids and fluids
\Addtotitle Proceedings of symposium in honour of %
R. S. Rivlin, Boston, Dec.~1987
Endref
\Newref COX89a
Cox \& Mortell*1989
\Names E. A. Cox*M. P. Mortell
\Article The evolution of resonant acoustic %
oscillations with damping
\Inbook Elastic wave propagation
\Frompage 173
\Topage 8
```



```

\Place Amsterdam
\Publisher Elsevir--North Holland
\Editor M. F. McCarthy*M. A. Hayes
Endref
\Newref MUR90a
Murphy*1990
\Names G. J. Murphy
\Book C*-algebras and operator theory
\Publisher Academic Press
Endref
\Newref OSE89a
\O~Searc\oid \& West*1989
\Names M. \O~Searc\oid*T. T. West
\Article Continuity of the generalized kernel and %
range of semi-Fredholm operators
\Journal Math.\ Proc.\ Camb.\ Phil.\ Soc.
\Volume 105
\Frompage 513
\Topage 22
Endref

```

We shall use the list above of references of this paper to illustrate the points to be borne in mind while preparing a reference file.

1. The first item in the file tells *MISTRESS* which syntax file to use; in this case it is *original.syn*. The extension must be *.syn* and is not written in.
2. The works should be listed in the order they are to be printed — alphabetical by first named author in the case of the Bulletin.
3. Each reference begins with the control sequence `\Newref` and ends with the word `Endref` followed by a line break.
4. There are essentially three different types of work: journal articles ([4], for example), books ([3], for example), and articles in books ([2], for example). In this last category we include books of conference proceedings with specific titles ([1] is an example). The three different types of work are



treated in exactly the same manner; only the nature of the information required changes.

5. Each item has a *MISTRESS* classification code. This should appear on the same line as the `\Newref` command, separated from that command by a space. Ideally every document should have its own distinctive classification code. The *MISTRESS* classification system allows for this. The first part of the classification code normally consists of what is called the `\Nickname`; the `\Nickname` is a sequence of letters of the alphabet, the first and last of which must be upper case; they would normally be the first three letters of the author's surname (I used MUR for [3]). Where there is any possibility of the complete classification code for two documents being identical, *MISTRESS* allows that this first part be enhanced by placing any number of lower case letters before the `\Nickname`; one possibility is to use initials for these lower case letters. This does not alter the `\Nickname`. The second part of the classification code consists of three symbols; the first two form what is called the `\Shortyear` — they will usually be the last two digits of the year of publication (use 00 for 1900 or 2000; use 01 for 1901, etc.), but if this is not appropriate two digits must appear anyway. The third character must be a letter; usually this will be lower case, but upper case can be used as well. In this way 52 different items can be classified as belonging to the same `\Nickname` and `\Shortyear` before it becomes necessary to add leading lower case letters to the classification codes. The *MISTRESS* classification code I used for Murphy's work is MUR90a.
6. On the line following `Newref`, there should appear what is called the `\Shortname`, then a single asterisk, then the `\Fullyear` followed by termination of the line. Note that in the first of our references `\Fullyear` was left empty, pending publication — hence the asterisk was the last character on the line. `\Shortname` and `\Fullyear` may be used by a syntax file either to refer to articles in the body of a document or to label them in the list at the end of the work. Their use



- depends on the wishes of the journal; `imsform` uses neither.
7. It is important that the classification code be reasonably easy to remember, because it is used to refer to articles in the body of a document. When I want to cite Murphy's book on C^* -algebras, for example, I type `\MUR90a*`. An optional parameter may appear between the classification code and the asterisk.
 8. The expansion of the command `\MUR90a*` will depend both on the syntax file in use and on the document being written. Had I wanted to refer to Murphy's section on the three fundamental results in K-theory I should have typed `\MUR90a, pages 229--40*`. The output would be: [3, pages 229-40]. The syntax file which decides this output for the IMS is `original.syn`. In that file, one finds the line `\CALLREF{[\Refcount\Parameter]}%` which explains the output. Note that the counter `\Refcount` is document dependent, since it refers to the position occupied by the article in the current list of references.

A different syntax file might use one or more of the macros `\Refcount`, `\Shortname`, `\Shortyear`, `\Fullyear` or `\Nickname`, or even a document dependent lower case letter accessed by `\Shortchar`. To explain this last macro, suppose that the list of references for an article contained three different works by Murphy dated 1990. Their classification codes might be `MUR90c`, `MUR90f` and `MUR90g`, say, in that order. Clearly the letters `c`, `f` and `g`, though used to cite the articles, would be meaningless for output in the context of the article. What `\Shortchar` does is to convert these letters into document dependent letters for the purposes of output. The `\Shortchar` for the three 1990 articles of Murphy would be `a`, `b`, and `c` respectively. If the syntax file in use specified `\CALLREF{[\Nickname\Shortyear\Shortchar\Parameter]}` then the expansion of, say, `\MUR90f, page 22*` would be `[MUR90b, page 22]`.
 9. Note that `\MUR90a` is not a control sequence; The control sequence is `\MUR` and the `90a` occur as parameters. It is necessary to mention this, because if the user is in the habit



- of creating short private macros whose names use upper case letters, he may encounter conflict. When using *MISTRESS* it is advisable to reserve upper case letters for reference use alone.
10. Every other item between `\Newref` and `Endref` starts with a control sequence and ends with a line break. The line break is being use as terminator for parameters; if it is desired to go onto a new line in the input file before the end of the parameter, then the comment marker (%) should be used to prevent \TeX from seeing the end of the line.
 11. Here is a list of the control sequences available:

<code>\Names</code>	<code>\Editor</code>	<code>\Book</code>	<code>\Edition</code>
<code>\Article</code>	<code>\Journal</code>	<code>\Volume</code>	<code>\Year</code>
<code>\Publisher</code>	<code>\Place</code>	<code>\Frompage</code>	<code>\Topage</code>
<code>\Inbook</code>	<code>\Status</code>	<code>\Addtonames</code>	<code>\Addtotitle</code>
<code>\Addtoeditor</code>	<code>\Addtovolume</code>	<code>\Addtoall</code>	

Any of the control sequences above which is applicable to the reference of a particular work may be used. The control sequences may occur in any order, with the single exception that `\Frompage` should appear somewhere before `\Topage`; a single page may be denoted by `\Frompage` alone.
 12. Note the inclusion of `\Year` in the list above. In fact, it is not necessary to use it provided `\Fullyear` has been specified, since the two will normally be the same. If `\Year` is used, its specification will supersede that which `\Fullyear` would have given to it. Note that the `\Year` will always be typeset using `\Yearfont`, whereas the syntax file may specify a different font for any particular use of `\Fullyear`.
 13. What should follow each of the control sequences can usually be inferred from the name. A few words must be said, however, about punctuation. The standard rule is that end-of-line punctuation should be omitted; the punctuation for the journal in question (in this case the IMS Bulletin) will be inserted automatically by a syntax file. In the case of journal names it is customary to abbreviate; if the last word of the journal name is shortened, the terminating full stop should be included. Note the line

`\Journal Math.\ Proc.\ Camb.\ Phil.\ Soc.`
in the reference for [4].

14. Tying together abbreviated words in journal names with a tilde can cause a difficulty in typesetting. In the case of surnames which consist of more than one word, it is essential to do so (note M. `\'0~Searc\'oid` in [4]). The reason for this is that some journals enter surnames before first names. *MISTRESS* will automatically reverse names if the journal style demands it, but must have some way of deciding what the surname actually is. It is defined to be the last continuous stream of tokens after the last space in the whole name. This leads to two other rules: names must always be entered with surname last; and surnames must be separated from first names or initials by a space. It also leads to the observation that T_EX-induced spaces in surnames must be suppressed; thus the surname Maol Mhichíl, which might ordinarily be typed `Maol~Mhich\'\'i 1` must in this context be typed without the space after `\i` (which removes the dot from the letter i). A satisfactory solution is `Maol~Mhich{\\'\'i}1`.
15. Where there are multiple authors or editors, they should be separated from one another not by standard punctuation but by a single asterisk.
16. The control sequence `\Names` refers to the headname or headnames in the reference, whether they denote authors, editors or translators.
17. The control sequence `\Editor` refers only to the editor or editors of the book in which the article in question is printed. This control sequence is not used, therefore, if the reference is to a complete book. Note that in [1] the author of the article, referred to using `\Names`, is the same as the editor, referred to using `\Editor`. Multiple editors are treated in exactly the same way as multiple names.
18. When the syntax file `original.syn` is in use, the abbreviation (ed.) or (eds) (depending on multiplicity) will always be typeset automatically after the `\Editor`.
19. Parentheses should not be put around the edition, year or volume or any other complete item of the description. If

these are required by the journal, they will be inserted automatically by the syntax file.

20. The control sequence `\Status` may be used to indicate that a work is 'to appear', or is in 'preprint' form, or whatever other description of that type is applicable. If the same work is to be referenced in journals of different languages, the T_EX command `\language` can be used to access various versions.
21. The control sequence `\Inbook` is used rather than `\Book` if the work being referred to is not the book but some article contained in it.
22. The five 'Addto' control sequences are included because it is sometimes necessary to give extra description of some part of a reference. Often this is typeset using a different font. For example a volume number may well not be sufficient, and a journal which demands a bold face 85 may well insist that it be followed by a roman (2). This would be dealt with by typing `\Addtovolume 2` somewhere in the description.
23. The control sequence `\Addtonames` always adds to `\Names` and never to `\Editor`. If any addition to `\Editor` is needed then the control sequence `\Addtoeditor` can be used.
24. The control sequence `\Addtotitle` similarly adds to `\Book` or to `\Inbook`, or, if no book is involved, to `\Article`. One obvious use of this is in the case of conference proceedings such as [1].
25. The text belonging to the control sequence `\Addtoall` is always typeset after everything else.
26. There is no necessity to complete a `.ref` file before starting an article; indeed it is highly unlikely that many authors would do so. *MISTRESS* allows dummy references to be collected in the main file as it is being written by using the control sequence `\Reftemp`. Here is how `\Reftemp` should be used: Suppose I had an empty `.ref` file when I was typing this article; then sometime after typing `\input mistress I` should have written (not necessarily all at once)


```
\Reftemp OSE89a MUR90a COX89a CAR87a ***
```

 (Note that the sequence of classification codes is terminated by *** followed by a line break.)

After the above use of `\Reftemp` I could use control sequences as described above to refer to [4], [3], [2] and [1] respectively in the body of my article, just as if entries for these articles were already present in my `.ref` file. These new entries are not in alphabetical order, so the numbers typeset by `TEX` would not be as they are in the final version, but at least `TEX` would not reprimand me for using an undefined control sequence and I would be able to get some idea of the general layout of the article without leaving spaces to be filled in later. If `TEX` is asked to typeset the list of references at the end of the paper, it will have a dummy entry corresponding to each article mentioned by `\Reftemp`, just as a reminder to the user that some references have not yet been properly defined. When ready to do so, the author can arrange the authors alphabetically and make sure that his `.ref` file contains the relevant entries. Then the `\Reftemp` command can be deleted from the file.

27. At the end of an IMS Bulletin document, the references and addresses are typeset using the single control sequence `\imsclose`. The heading and all spacing are included in this macro. In general the *MISTRESS* control sequence `\References` would be used to typeset references alone, but it is included as part of the `\imsclose` macro.
28. The default heading given in `original.syn`, the syntax file for the Bulletin, for a list of references is `References`; this is automatically changed to `Reference` if there is only one item. If a list is better described as a `Bibliography` or as something else, then the heading can be altered by typing `\def\Refheadtext{Bibliography}` or whatever else is appropriate somewhere in between `\input mistress` and `\References`.
29. The maximum number of references that can be dealt with by *MISTRESS* is large, and obviously will depend on how many other macro packages are being used. This article uses quite an extensive package from THE GRAFPAK as well as the *MISTRESS* macros. I typeset it with a list of 640 references as a test, and there was plenty of memory left from a

standard version of `sbTEX`. It should be said, however, that the *MISTRESS* system is deliberately not the most efficient it is possible to build, because the emphasis has been on adaptability and ease of use. Consequently each reference may construct from one to three macros simply so that the user can recall it in the body of an article. After a few hundred such assignments memory allocations could become scarce if `TEX` is dealing with a lot of other input matter. One way of saving both time and a large amount of memory has been built in by allowing the user to specify that he does not want to refer to any work in the list. In long lists such as bibliographies, this may well be the case. To by-pass the calling mechanism, the user should type `\let\Refcalling\relax` before writing `\input mistress` near the head of the file. Please do not use this by-pass in articles for the Bulletin.

Letting *MISTRESS* write your files

Most of us use `TEX` only for typesetting, but this marvellous system of Knuth's is capable of doing many other things for which we might normally use one of the standard multi-purpose programming languages. Reading and writing of files is one such function.

No user interface has yet been given to *MISTRESS* for the automatic writing of reference lists in the manner prescribed above; perhaps that will be included in the next version — it could even be done entirely in `TEX`. However, *MISTRESS* will instruct `TEX` to do something which is perhaps more complicated (in `TEX` terms), namely to pull out of several master reference files all the references needed for a particular article and write them into a special `.ref` file to go with that article. This is done by using the control sequence `\Refswanted`. Near the top of this file, for example, after `\input mistress` there is the following command:

```
\Refswanted **{original} *{c} COX89a CAR87a
*{m} MUR90a *{o} OSE89a ***
```

By way of explanation:

1. Each of the references is specified by its classification code (flanked by spaces), exactly as it would have been when using `\Reftemp`.

2. Immediately after the `\Refswanted` command there must be a space and two asterisks. These are followed immediately by the name of the syntax file, without the `.syn` extension, flanked in curly brackets and followed by a space.
3. Then a single asterisk precedes the name of the master reference file (in curly brackets) from which the next batch of references are to be read. The extension must be `.ref` and must not be written here. In this case the file is `c.ref` and in it I keep all references to articles whose first named author has surname beginning with the letter C. Note that later on the command `*{m}` occurs, telling `TeX` to read instead from the file `m.ref`; later on again, `TeX` is told to move to `o.ref`.
4. After the master file has been specified, there comes a list of the classification codes of all articles whose reference structure is to be pulled out of that file. These can be in any order; the order of typesetting will be determined by the order in which they occur in the master file.
5. The whole is completed by typing `***` followed by a line break.
6. Note that the master files are listed alphabetically, so that Cox and Carroll (not in that order) will be listed before Murphy, and Murphy before Ó Searcóid.
7. `TeX` will be informed by *MISTRESS* how many references are to be pulled out of any given master file. The procedure followed then is that `TeX` will start reading that file; every time `TeX` sees `\Newref`, it will look at the classification code which follows. If it recognizes it as being wanted, `TeX` will read on to `Endref` and will copy out the whole of the reference into the new file. If `TeX` sees that the particular reference is not one of those wanted, it will ignore most of the text down to the next `Endref` (to be precise, `TeX` will at this stage be interpreting the backslash as a comment character; since most lines of a *MISTRESS* reference begin with a backslash, this means that `TeX` will not even read them). `TeX` will carry on in this way until it has got hold of the number of references asked for by *MISTRESS*. It will then stop reading and immediately close up the master file.

8. It is still the practice of some journals to list references at the end of an article in the order in which they were cited, rather than in alphabetical order. This can be handled by *MISTRESS*, though more time is involved, by making sure that each classification code in the list of `\Refswanted` is preceded by an indication of which reference file it should be taken from. This practice ensures that the order exhibited in the `\Refswanted` list is the same as the order in the `TeX`-written `.ref` file. For this paper I should have typed


```
\Refswanted **{original} *{o} OSE89a
*{m} MUR90a *{c} COX89a *{c} CAR87a ***
```

 Note that `TeX` knows when there is only one item to be located in any given master file and will not read beyond it. It is wise, however, to make sure that master files do not contain an unwieldy number of references.
9. On `TeX`'s first run, a file whose name is identical to that of the main file, but whose extension is `.ref` will be written according to the specifications given by `\Refswanted`, provided no such file exists. If such a file exists, as it will do on subsequent runs, the command `\Refswanted` will either be ignored or will be used only to get information from the `.ref` file. All the information between the control sequence and the terminating three asterisks will be skipped over.
10. One may use `\Reftemp` in conjunction with `\Refswanted`, but note that numbering may be temporarily upset by `\Reftemp`.
11. `\Refswanted` should not be deleted after it has been used. When the final version of the paper is being written, new items which had been only temporarily listed with `\Reftemp` can be added to their correct master files, and their classification codes transferred from `\Reftemp` to `\Refswanted`. Then, if the `.ref` file specific to the document is erased, a further run of `TeX` will rewrite it with all the new items included.

Arrangement of master files

On referring to an article or book for the very first time, one writes the standard *MISTRESS* reference entry for that work; one then presumably tests it to make sure that no mistakes have

been made. The reference may be required again at some future date, so it should be filed away in some master file of references. It makes sense that it should go into its correct alphabetical position in that master file. Over the years, however, one's catalogue of references is liable to increase substantially and it is reasonable that several such master files should be kept. This is advisable also in view of the fact that the amount of reading which T_EX has to do will, in the long term, be less if the complete library of references is split up. So a classification problem arises.

Retrieval and copying must be the key elements in the arrangement of master files. Each master file should certainly be arranged alphabetically, but one must still decide which master file any particular reference should go into. It is worth while keeping the following points in mind:

1. If partitioning by subject or by some similar criterion is required, it would be well to make sure that this is not so detailed as to separate into different categories any two articles which may be referred to in the same document. It would be unwise, for example to have one set of master files for Functional Analysis and another for Operator Theory, since that would create minor difficulties in ensuring that the .ref file for any document which referred to articles in both these categories would itself be automatically alphabetically ordered. Having separate categories for Celtic Studies and Numismatics might however be permissible.
2. The lowest level of file arrangement should be alphabetic. Suppose there is a category of master files for Pure Mathematics. There could be a great many articles cited in these master files, but it is perhaps best that they be separated alphabetically. One file may contain the works of authors whose surnames begin with A, B or C, say, and the next those whose names begin with D or E, and so on. If this is done then any list of references which cites only articles in Pure Mathematics can be arranged automatically in alphabetical order without T_EX having to look at any master file more than once.

Syntax files

MISTRESS cannot operate without a syntax file. When a syntax file is specified by `\Refswanted` then the line `\Syntax {<filename without extension>}` will appear at the head of the .ref file that T_EX writes for the document. If the user writes his own .ref file then he must write in a similar way the name of the .syn file (without the extension .syn which will be assumed) which *MISTRESS* should call up.

Syntax files contain information about the different fonts to be used at different parts of a citation, about the punctuation to be used, about the mode of reference to be adopted in the body of an article, about the labelling of references in the list at the end of an article and about one or two other minor details. It is hoped to develop a library of syntax files for different journal styles; these can never of course be authoritative without the approval of the editorial staff of a journal. If care is taken, however, to copy exactly the style actually in use by that journal, there is every reason to suppose that the output will be acceptable.

Here is the text of the original .syn file which we shall use as an example.

```
\REFHEADING{\centerline{\bf\Refheadtext}\smallskip
\Pointsize\Setfonts\emergencystretch=13pt}%
\def\Refheadtext{\ifnum\Refcount>1 References\else
Reference\fi}%
\REFINDICATOR{\hbox to -7pt{\hss[\Refcount]}\kern7pt}%
\CALLREF{[\Refcount\Parameter]}%
\def\INTERNAMES{,}%
\def\And{\ and}%
%
\IFINJOURNAL
*\NAMES
\ * \ADDTONAMES
, * \TITLE
, * \JOURNAL{\ }
\ * \VOLUME
(* \ADDTOVOLUME)
\ (* \YEAR)
```



```

, *\PAGE
\quad(*\STATUS)
\rm. *\empty
*\ADDTOALL.
***
%
\IFINBOOK
*\NAMES
| *\ADDTONAMES
, *\TITLE
|/\bf\ in *\INBOOK
| (*\EDITION)
. *\ADDTOTITLE
, *\EDITOR\ (\ifnum\Plural>1 eds\else ed.\fi)
, *\ADDTOEDITOR
, *\PUBLISHER
: *\PLACE
, *\YEAR
, *\PAGE
\quad(*\STATUS)
\rm. *\empty
*\ADDTOALL.
***
%
\IFBOOK
*\NAMES
| *\ADDTONAMES
, *\TITLE
| (*\EDITION)
, (*\ADDTOTITLE)
. *\PUBLISHER
: *\PLACE
, *\YEAR
\quad(*\STATUS)
\rm. *\empty
*\ADDTOALL.
***

```



```

%
\def\Setfonts{\rmFonts\Namesfont\Editorfont\Bookfont
\Inbookfont\Journalfont\Yearfont\Publisherfont
\Placefont\PageNumberfont\Statusfont\Addtonamesfont
\Addtotitlefont\Addtoallfont\Editionfont
\Addtovolume\Addtoeditorfont*\slFonts\Articlefont*
\bfFonts\Volumefont*} \Reversenames=0
\Reverseednames=0
\Frenchspacing=0

```

The various parts of this file are explained below:

1. The first item tells us that the reference list should be headed by whatever is contained in the macro `\Refheadtext`; that this should be printed in bold face in the centre of a line, and that it should be followed by a small vertical skip. The control sequence `\Pointsize` is set to `\relax` (i.e. do nothing) by *MISTRESS* unless it has previously been defined. In fact, if one is writing for the IMS and `imsform` was input before `mistress`, then `\Pointsize` will have been defined to be `\ninepoint`. This means that the reference list will be written using fonts of nine point size and with appropriate spacing. This decreases the likelihood of overfull hboxes which is bound to be a difficulty when dealing with a rather narrow page.

The `\Setfonts` control sequence sets all the fonts necessary for different parts of each citation. It will be explained further on.

The use of `\emergencystretch` needs some explanation. It may not be familiar to some users of `TEX` since it did not exist before `TEX3`. `TEX` does not in general allow too much freedom in word spacing because it wants the output to look as good as is possible; it hyphenates, if that is allowed by its hyphenation table, rather than allow too much ugly white space to appear on a line. It does happen, however, particularly when the line width is narrow, that `TEX` is incapable of either hyphenating correctly or spacing according to the rules laid down; in such cases we are likely to encounter underfull and overfull hboxes. One partial solution to this is `\hfuzz`



which allows text to stick out beyond the end of a line, but this can look ugly even if the jut out is only two or three pixels. The new macro `\emergencystretch` allows \TeX to insert more white space in cases where no other solution is possible. It is used in the reference section of this package to try to ensure that the user never need tamper with the text of references in order to avoid `\hbox` difficulties.

If you are using any version of \TeX prior to \TeX 3 you should get hold of the most up to date version and destroy your earlier one. In case any of our readers is unaware of the fact, `sbTeX` is a fast, efficient and compact version of \TeX for the PC; it is one of the few versions which is kept completely up to date by its author, Wayne Sullivan of the UCD Mathematics department, and it is available free of charge.

2. The second item tells \TeX what the `\Refheadtext` should normally be, namely the word `References` unless the number of references (recorded by `\Refcount`, which at this stage of the operation will return the full number of references) is 1, in which case the singular version is used. If alteration is required, this should be done in the main document as was described in an earlier section.
3. Next `\REFINDICATOR` tells \TeX both how to label references in the list and where to put the label. In the original syntax file the indicator is the number of the reference, accumulated by `\Refcount`, inside square brackets. It is placed in the left hand margin with its right edge a distance of 7pt from the start of the text. Other possibilities for `\REFINDICATOR` might include the use of `\Shortname`, `\Shortyear`, `\Shortchar`, `\Fullyear`, or `\Nickname` as well as or instead of `\Refcount`. Note that this makes the popular Harvard style possible.
4. Then `\CALLREF` tells \TeX how citations in the body of an article should be made. This syntax file says that a citation should consist of the number of the reference followed by any parameter included in the text of the document (the termination of which is indicated there by an asterisk), the whole enclosed in square brackets. It has been explained in the last section how a syntax file can be set up to produce



quite different citations.

5. `\INTERNAMES` tells \TeX what punctuation to use in between names of multiple authors or editors; `\And` takes the place of `\INTERNAMES` before the last name in such a sequence.
 6. Next come three sequences which describe how the three different types of reference should be typeset. These begin with the control sequences `\IFINJOURNAL`, `\IFBOOK` and `\IFINBOOK` respectively, each written on a line by itself. Each is terminated by a three consecutive asterisks on a line by themselves. In between the control sequence and the three stars there are several lines of instruction. These lines must all conform to a single pattern. They begin with some sequence of text (usually punctuation) terminated by a star. A single control sequence follows. After that there is more text terminated by a line break. Either or both of the two sequences of text may be empty. The rule that `MISTRESS` tells \TeX to follow is this: if the control sequence has been set to `\relax` then the whole line is ignored. Otherwise the macro will be expanded and the whole line will be typeset. Each of the control sequences involved here is initially set to `\relax` for each work listed; that is changed only if their is an appropriate entry in the `.ref` file for that work; for example `\Names` will change the relaxed status of `\NAMES` and will decide what text forms the expansion of `\NAMES`; `\Book` or `\Inbook` or `\Article` decides what goes into `\TITLE`, `\Publisher` decides what goes into `\PUBLISHER`, `\Frompage` and `\Topage` decide what goes into `\PAGE` and so on. Note that a trailing space should be made explicit before the line break or it may not be noticed. This was done after `\JOURNAL` above.
- Also, if one wants to insist that certain punctuation be typeset independently of the expansion of any `MISTRESS` macro then the \TeX control sequence `\empty` can be used since it is never equivalent to `\relax`, but it expands to nothing (it is used above in `original.syn`).
7. Note that `\Names` are automatically counted by `\Plural`, so that it can be specified in the syntax file how multiple authors might be dealt with differently from single ones. `\Plural` is



reset to 0 by `\Editor` and the names for this command are similarly counted. `original` makes use of `\Plural` to specify that an editor is denoted by (ed) but that multiple editors are denoted by (eds).

8. All fonts for citations must be specified in the definition of `\Setfonts` in the syntax file. These are the fonts:

<code>\Namesfont</code>	<code>\Editorfont</code>	<code>\Bookfont</code>
<code>\Inbookfont</code>	<code>\Journalfont</code>	<code>\Yearfont</code>
<code>\Publisherfont</code>	<code>\Placefont</code>	<code>\Pagefont</code>
<code>\Statusfont</code>	<code>\Addtonamesfont</code>	<code>\Addtotitlefont</code>
<code>\Addtoallfont</code>	<code>\Editionfont</code>	<code>\Addtovolumefont</code>
<code>\Articlefont</code>	<code>\Volumefont</code>	<code>\Addtoeditorfont</code>

If fonts are to be non-standard, then the fonts should be initialized in the usual way. For those which are to be simply `\rm`, `\it`, `\bf` or `\sl` *MISTRESS* provides a short way of setting them. For example

```
\rmFonts\thisfont\thatfont\totherfont*
```

sets those three control sequences to be globally identical to the current meaning of `\rm` (in a reference list for the Bulletin, that is nine point computer modern roman). Note the asterisk which terminates the use of `\rmFonts`.

9. Some journals insist on reversing the names of authors. The Proceedings of the Royal Irish Academy, for example, follows that practice. That journal does not, however, reverse the names of editors. *MISTRESS* provides two numbers `\Reversenames` and `\Reverseednames` which are set to zero by default. If initials (or first names) are to appear after the surname, then either or both of these numbers should be given a positive value. Where reversal is effected, a comma will separate the surname from what follows.
10. If frenchspacing is required in the references, then this can be indicated in the syntax file by saying `\Frenchspacing=1`. If frenchspacing is turned on in this way for references, it will be turned off again afterwards; otherwise any frenchspacing operative independent of *MISTRESS* will not be tampered with.
11. All syntax files made for public use should have the extension



`.syn`. They should be clearly marked as being for use with *MISTRESS*, and should state the name and address of the author together with the date of the last amendment.

For the purposes of comparison, the references for this paper are listed twice using the same `.ref` file, first with the syntax file `riaproc.syn`, which approximates the specifications for the Proceedings of the Royal Irish Academy in so far as that is possible on this narrow page, then with the syntax file `original.syn`.

Notes for editors

The editor or production manager should have, in addition to the files `imsform.tex`, `mistress.tex` and `original.syn`, the following four files: `imslogo.tfm`, `imslogo.pk`, `imsbegin.tex` and `imsvar.tex`.

The first two files are for printing the logo of the Society. The `.tfm` file should be stored in the usual place where `TeX` looks for such files, and the `.pk` file, which is for use with 300 dpi printers, should be stored where `TeX` looks for such files. Depending on the directory structure used on the machine, it may be necessary to change the extension of the file name from `.pk` to `.300`.

The font `\imslogo` contains the logo of the Society in 26 different sizes; the smallest, `\char1`, appears on page headlines in this issue and the largest, `\char26`, is that appearing on the cover.

The file `imsbegin.tex` typesets the cover, inside front cover, contents page, and the pages numbered with roman numerals. It should be placed in the directory where `TeX` looks for input files. Changes in `imsbegin.tex` will be necessary only if the editor decides to change the format or wording of the material. Changes of fees and of officers are provided for in `imsvar.tex`.

The file `imsvar.tex` must be in the directory where typesetting of the current issue of the Bulletin is taking place. It contains all the variables for `imsbegin` which change from time to time, such as names and addresses of officers and membership fees. This file should be read and kept up to date by the editor.

To typeset an issue of the Bulletin, the editor should make sure first of all that each of the files for inclusion is to his satis-



faction. He should ensure that each file contains an `\imscontadd` instruction for inclusion in the contents page, and that each file is terminated by `\bye`. He should check that the `\imsshorttitle` is correct for each file. Then he should collect all the files for inclusion and the input files they call on in the current directory together with the current version of `imsvar.tex`.

The main file should then be written. The main file for the current issue of the Bulletin is as follows:

```
\input imsform
\imsnumber=27
\input mistress
\imsinput minsa91
\imsinput sep91
\imscontadd{Thesis Abstracts&&}
\imsinput aisling
\imsinput susan
\imscontadd{Research Announcement&&}
\imsinput chris
\imscontadd{Directions for Contributors&&}
\imsinput ti
\imscontadd{Articles&&}
\imsinput 1891
\imsinput reid
\imscontadd{Book Review&&}
\imsinput wardrev
\imsinput imsbegin
\bye
```

Note the following:

1. Both `imsform` and `mistress` should be input in the main file, even though they may be input again in the individual files. At the stage of a second input, `TeX` will not bother re-reading macros it has already stored.
2. `\imsnumber` should be set equal to the number of the issue. The month and year will be calculated automatically. If for any reason the calculation becomes incorrect, the editor should follow the instructions given in `imsvar.tex` for



setting them right.

3. Several instances of `\imscontadd` occur in this file. In each of them, the entry after the second ampersand is empty. In this case, `TeX` has been instructed to leave some space and typeset the title entry in bold face.
4. `\imsinput` is used instead of `\input` for each of the articles to be input. This ensures, amongst other things, that `\bye` will not cause `TeX` to stop typesetting.
5. `\bye` in the main file has its usual meaning.
6. The file `imsbegin` is listed last, after the contents have been built up. It will arrange for the automatic typesetting of a contents page, amongst other things. There is a very slight possibility of error in the roman numerals in the list of contents which indicate the pages on which the list of officers and the notes on membership occur. If this happens, then the editor should type `\imsstartpage=x` where x denotes the number (as an Arabic numeral, not roman) of the page on which the list of officers is to be found.
7. The number of pages before that numbered with the arabic numeral 1 should always be even. If it is not, then an extra blank page should be inserted before sending the material to the printer. Failure to do this could result in an issue like number 25 where odd-numbered pages were on the left and even-numbered on the right.
8. When the complete issue has been typeset and printed, the editor should move to the front those pages of `\imsbegin` which belong there before sending the package to the printer.

Final notes

1. Wayne Sullivan has arranged for the files `imsform.tex`, `mistress.tex`, `original.syn` and `riaproc.syn` to be resident on his MATHDEP noticeboard. They can be picked up from there by sending a message to `LISTSERV`. On your mainframe, you should e-mail to `listserv @irlearn.ucd.ie` and include the following four lines as the body of the message:



```
get imsform tex
get mistress tex
get original syn
get riaproc syn
```

Note that this message is to be sent to LISTSERV, not to MATHDEP.

2. These files are copyright and should not be altered. The *MISTRESS* package is free to individual members of the IMS, only on condition that any syntax files written by members are sent to me for free distribution with *MISTRESS*. `original.syn` is the syntax file for the Bulletin; `riaproc.syn` attempts to imitate the syntax for the Proceedings of the RIA.
3. The files `imsform.tex`, `mistress.tex` and all `.syn` should be placed in a directory where \TeX looks for input files.
4. Authors are reminded that their own private macros for a paper should either be included in the \TeX source file or, if written in a separate input file, should accompany that file when it is submitted to the Bulletin.
5. When a paper is written in \TeX for the Bulletin, two printed copies should be sent by mail to the editor and the \TeX input file should also be sent in the manner described on the inside of the back cover of the most recent issue of the Bulletin.

Acknowledgement

The Mathematics department in UCD is fortunate in having its own resident \TeX wizard in Wayne Sullivan. We are also fortunate in that he is usually willing to drop what he is doing and help solve other people's problems on the spot. I have availed of his support on countless occasions; without it, the work of devising these files would have been much more difficult and they would certainly have contained more errors than they do now.



REFERENCES

- [1] CARROLL, M. M. (in press) Exact solutions in compressible isotropic elasticity. In M. M. Carroll (ed.), *Nonlinear effects in solids and fluids*. Proceedings of symposium in honour of R. S. Rivlin, Boston, Dec. 1987.
- [2] COX, E. A. and MORTELL, M. P. 1989 The evolution of resonant acoustic oscillations with damping. In M. F. McCarthy and M. A. Hayes (eds), *Elastic wave propagation*, 173-8. Amsterdam. Elsevir-North Holland.
- [3] MURPHY, G. J. 1990 *C*-algebras and operator theory*. Academic Press.
- [4] Ó SEARCÓID, M. and WEST, T. T. 1989 Continuity of the generalized kernel and range of semi-Fredholm operators. *Math. Proc. Camb. Phil. Soc.* 105, 513-22.

References

- [1] M. M. Carroll, *Exact solutions in compressible isotropic elasticity in Nonlinear effects in solids and fluids*. Proceedings of symposium in honour of R. S. Rivlin, Boston, Dec. 1987, M. M. Carroll (ed.) (in press).
- [2] E. A. Cox and M. P. Mortell, *The evolution of resonant acoustic oscillations with damping in Elastic wave propagation*, M. F. McCarthy and M. A. Hayes (eds), Elsevir-North Holland: Amsterdam, 1989, 173-8.
- [3] G. J. Murphy, *C*-algebras and operator theory*. Academic Press, 1990.
- [4] M. Ó Searcóid and T. T. West, *Continuity of the generalized kernel and range of semi-Fredholm operators*, *Math. Proc. Camb. Phil. Soc.* 105 (1989), 513-22.

Mícheál Ó Searcóid,
Roinn na Matamaitice,
Coláiste na hOllscoile,
Baile Átha Cliath.

100 YEARS OF DIXON'S IDENTITY

James Ward

Although Queen's College, Galway cannot boast of such a world-class mathematician as Boole (Queen's College, Cork), nevertheless if one considers the size of Galway, the relative poverty of the hinterland and the remoteness of Galway from other centres of learning, mathematicians such as Allman, Dixon and Bromwich who each held the chair of mathematics at QCG were of a very high calibre indeed.

Perhaps the most distinguished of this trio was Dixon (though Bromwich would have his admirers too) who was appointed to the Chair in Mathematics at Queen's College, Galway in 1893 and to the Chair in Mathematics at Queen's College, Belfast in 1901, where he remained until his retirement in 1930. A broad account of Dixon's life and work can be found in the obituary written by E. T. Whittaker [6] from which the following biographical information has been extracted.

Alfred Cardew Dixon was born on the 23rd May, 1865 at Northallerton, Yorkshire, went up to Trinity College, Cambridge as a major scholar and in the Tripos of 1886 ("an exceptionally strong year" — Whittaker [6]) graduated as Senior Wrangler. He was awarded a Smith's Prize in 1888 and elected a fellow of Trinity College in the same year. In [6] Whittaker notes that in his early years Dixon had produced comparatively little work of real distinction, but that from 1893 (the year he was elected to the Chair in Mathematics at Queen's College, Galway) "he became a most productive original worker".

Dixon produced important memoirs on ordinary and partial differential equations, Abelian integrals, automorphic functions, Fredholm theory and functional equations. He was elected to

Fellowship of the Royal Society in 1904 and was President of the London Mathematical Society in 1931–33 following his retirement from Queen's University, Belfast. Dixon died on 4th May 1936 and had been predeceased by his wife in 1926; they had no children.

1991 marks the 100th anniversary of the appearance of a note by Dixon [1] proving the combinatorial identity

$$(*) \quad \sum_{k=0}^n (-1)^k \binom{n}{k}^3 = \begin{cases} \frac{(-1)^m (3m)!}{(m!)^3} & \text{if } n = 2m \\ 0 & \text{otherwise.} \end{cases}$$

In the literature on combinatorial theory this identity or the following slight generalisation due to Fjeldsted [3] (see also Dixon [2]) now bears Dixon's name:

$$\sum_k \binom{a+b}{a+k} \binom{b+c}{b+k} \binom{c+a}{c+k} (-1)^k = \frac{(a+b+c)!}{a!b!c!}$$

for integers $a, b, c \geq 0$ and the permitted range of the integer k (which is finite).

There are several proofs of Dixon's identity and I wish to present three such proofs, namely Dixon's original proof, a second using the Lagrange inversion formula as described in [4] and a third using WZ pairs [7, p.126]. These illustrate the rich diversity of techniques in Combinatorics, but I would also like to draw attention to the fine mathematical heritage of Queen's (now University) College, Galway as represented in the work of Allman, Bromwich, Dixon and others.

First Proof (Dixon [1])

For n a positive integer, and writing $(1+x)^n$ as

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

(here the a_i are the binomial coefficients $\binom{n}{i}$) it follows that

$$(**) \quad a_0^2 - a_1^2 + a_2^2 - \dots - a_n^2$$

is zero if n is odd, s an arbitrary positive integer (${}^n C_i = {}^n C_{n-i}$), so the question is to find a closed form for such a sum (***) when n is even. When n is even, say $n = 2m$, and $s = 3$, we denote by S the alternating sum (***) so obtained, and in this case S is the left hand side of the formula (*). Now S is the coefficient of $y^{4m} z^{4m}$ in $(1 - y^2)^{2m} (1 - z^2)^{2m} (1 - y^2 z^2)^{2m}$, which is the term independent of y and z in

$$(y - y^{-1})^{2m} (z - z^{-1})^{2m} (yz - y^{-1} z^{-1})^{2m}.$$

Making the trigonometric substitutions $y = \cos \theta + i \sin \theta$ and $z = \cos \phi + i \sin \phi$, S becomes the absolute term in

$$(-4)^{3m} \sin^{2m} \theta \sin^{2m} \phi \sin^{2m} (\theta + \phi)$$

when expanded in cosines of multiples and sums of multiples of θ and ϕ , so (1)

$$(-1)^m 4^{-3m} 4\pi^2 S = \int_0^{2\pi} \int_0^{2\pi} \sin^{2m} \theta \sin^{2m} \phi \sin^{2m} (\theta + \phi) d\theta d\phi.$$

The Binomial Theorem is used to expand $\sin^{2m} (\theta + \phi)$ via $(\sin \theta \cos \phi + \cos \theta \sin \phi)^{2m}$, every second term in the integral vanishes and the right hand side of (1) reduces to

$$\sum_{k=0}^m {}^{2m} C_{2k} \int_0^{2\pi} \sin^{2(m+k)} \theta \cos^{2(m-k)} \theta d\theta \cdot \int_0^{2\pi} \sin^{2(2m-k)} \phi \cos^{2k} \phi d\phi.$$

Each integral is 4 times the integral over 0 to $\pi/2$. A further substitution of $x = \sin^2 \theta$, $y = \sin^2 \phi$ results in an integral of the type

$$\int_0^1 x^{\alpha-1} (1-x)^{\beta-1} dx \cdot \int_0^1 y^{\gamma-1} (1-y)^{\delta-1} dy$$

which is a product of Beta integrals, equal to $\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} \cdot \frac{\Gamma(\gamma)\Gamma(\delta)}{\Gamma(\gamma+\delta)}$.

Using $\Gamma(\alpha) = (\alpha-1)\Gamma(\alpha-1)$, and $\Gamma(1/2) = \sqrt{\pi}$, and some diligent calculations, the $4\pi^2$ term in the left hand side of (1) cancels and we are left with

$$\sum_{k=0}^m \frac{(2m)!}{(2k)!(2m-2k)!} \times \frac{1 \cdot 3 \cdot 5 \cdots (2m+2k-1) \cdot 1 \cdot 3 \cdot 5 \cdots (2m-2k-1)}{2 \cdot 4 \cdot 6 \cdots 4m} \times \frac{1 \cdot 3 \cdot 5 \cdots (4m-2k-1) \cdot 1 \cdot 3 \cdot 5 \cdots (2k-1)}{2 \cdot 4 \cdot 6 \cdots 4m}$$

on the right hand side of (1), which simplifies (!) to

$$\frac{(4m)!}{2^{8m} (m!)^2 (2m)!} \left\{ 1 + m \frac{2m+1}{4m-1} + \frac{m(m-1)}{2} \frac{(2m+1)(2m+3)}{(4m-1)(4m-3)} + \cdots (m+1 \text{ terms}) \right\} \quad (2)$$

Dixon now refers to Wolstenholme's Problems 2nd edition #303 wherein

$$1 - m \frac{a}{b} + \frac{m(m-1)}{2!} \frac{a(a-1)}{b(b-1)} - \frac{m(m-1)(m-2)}{3!} \frac{a(a-1)(a-2)}{b(b-1)(b-2)} + \cdots (m+1 \text{ terms})$$

is equal to

$$\left(1 - \frac{a}{b}\right) \left(1 - \frac{a}{b-1}\right) \left(1 - \frac{a}{b-2}\right) \cdots \left(1 - \frac{a}{b-m+1}\right)$$

and (2) falls into this pattern on putting $a = -m - 1/2$ and $b = 2m - 1/2$ which finally results in (*).

Perhaps this clever proof justifies Whittaker's remark [6] "if the method was possible Dixon would make it work".

Second Proof (Goulden-Jackson [4, p.23-4])

This requires some preliminary notation. We denote by $R[[t]]$ the ring of formal power series in a variable t over a commutative ring

R , so an element f of $R[[t]]$ is of the form $f = \sum_{k \geq 0} a_k t^k$, $a_k \in R$. If we allow finitely many negative powers of t , we have the ring of (formal) Laurent series $R((t))$. A polynomial is an element of $R[[t]]$ with only finitely many non-zero a_k . Two important subsets of $R[[t]]$ are

$$R[[t]]_0 = \{f \in R[[t]] \mid a_0 = 0\} \quad \text{and}$$

$$R[[t]]_1 = \{f \in R[[t]] \mid a_0^{-1} \text{ exists}\};$$

in this latter case f has an inverse in $R[[t]]$ (long division!). We let f' denote the formal derivative of f which is $\sum_{k \geq 0} (k+1)a_{k+1}t^k$, and define the "coefficient operator" $[t^k]f$ to pick out the coefficient of t^k in f , thus $[t^k]f = a_k$ and $[t^k]f' = (k+1)a_{k+1}$. Finally if $f_1 \in R((t))$, the valuation of f_1 is defined to be

$$\text{val}(f_1) = \begin{cases} k & \text{if } f_1(t) = t^k g(t), g(t) \in R[[t]]_1 \\ \infty & \text{otherwise.} \end{cases}$$

Now we can state Lagrange's Inversion Formula. Let $\phi(t) \in R[[t]]_1$. Then there exists a unique formal power series $w(t) \in R[[t]]_0$ such that $w(t) = t\phi(w(t))$. Moreover if $f(x) \in R((x))$ (Laurent series) then (a)

$$[t^n]f(w) = \begin{cases} \frac{1}{n}[x^{n-1}]\{f'(x)\phi(x)^n\} & \text{for } n \neq 0, n \geq \text{val}(f) \\ [x^0]f(x) + [x^{-1}]f'(x) \log(\phi(x)\phi(0)^{-1}) & \text{for } n = 0 \end{cases}$$

$$[\exp(x) = \sum_{j \geq 0} \frac{x^j}{j!} \in R[[x]], \quad \text{and} \quad \log(\exp(x)) = x].$$

If $F(x) \in R[[x]]$ then (b)

$$\sum_{n \geq 0} c_n t^n = F(w) \{1 - t\phi'(w)\}^{-1} \quad \text{where } c_n = [x^n]\{F(x)\phi(x)^n\}.$$

Example

Suppose we want to invert $w(t) = te^w$, ie to express w as a power series in t , we have $\phi(t) = e^t$, $f(w) = w$, and $\text{val}(f) = 1$; so by (a)

$$[t^0]w(t) = [x^0]f(x) + [x^{-1}]1 \cdot \log(e^x \cdot 1) = 0$$

$$\text{and} \quad [t^n]w(t) = \frac{1}{n}[x^{n-1}]\{1 \cdot e^{nx}\} = \frac{1}{n}[x^{n-1}]\left\{\sum_{k \geq 0} \frac{(nx)^k}{k!}\right\}$$

$$= \frac{1}{n} \frac{n^{n-1}}{(n-1)!} = \frac{n^{n-1}}{n!}$$

hence $w(t) = \sum_{n \geq 1} (n^{n-1}/n!)t^n$, note that $w(t) \in R[[t]]_0$. Lagrange's formula will also give power series expansions for $w^{-1}(t)$ ($f(w) = w^{-1}$) or $w^{-2}(t)$ ($f(w) = w^{-2}$) etc. ■

There is a multivariate version of the Lagrange formula [4, p.21] and the multivariate version of (b) reads: If

$$F(\mathbf{x}), \phi_1(\mathbf{x}), \dots, \phi_m(\mathbf{x}) \in R[[\mathbf{x}]] \quad \text{where } \mathbf{x} = (x_1, \dots, x_m),$$

and if

$$w_i = t_i \phi_i(\mathbf{w}) \quad \text{for } 1 \leq i \leq m \quad \text{where } \mathbf{w} = (w_1, \dots, w_m),$$

then

$$\frac{F(\mathbf{w})}{\det(\delta_{ij} - t_i \partial \phi_i(\mathbf{w}) / \partial w_j)} \Big|_{\mathbf{w}=\mathbf{w}(t)} = \sum_{\mathbf{k} \geq 0} t^{\mathbf{k}} [\mathbf{x}^{\mathbf{k}}] \{F(\mathbf{x}) \phi(\mathbf{x})^{\mathbf{k}}\}$$

where $\mathbf{k} = (k_1, \dots, k_m)$.

For instance suppose we have $X_i = \sum_{j=1}^m a_{ij} x_j$ and we want to find the coefficient of $\mathbf{x}^{\mathbf{k}}$ in $\mathbf{X}^{\mathbf{k}}$, ie the coefficient of $x_1^{k_1} \dots x_m^{k_m}$ in $X_1^{k_1} \dots X_m^{k_m}$. We apply the above with $F(\mathbf{x}) = 1$, $\phi_i(\mathbf{x}) = a_{i1}x_1 + \dots + a_{im}x_m$ so the ϕ_i are linear functions, to get that the coefficient of $\mathbf{x}^{\mathbf{k}}$ in $\mathbf{X}^{\mathbf{k}}$ is equal to the coefficient of $\mathbf{x}^{\mathbf{k}}$ in $|I - x_i a_{ij}|^{-1}$ where I is the $m \times m$ identity matrix. If we put $A = (a_{ij})$ and $X = \text{diag}(x_1, \dots, x_m)$ the formula reads as the coefficient of $x_1^{k_1} \dots x_m^{k_m}$ in $|I - XA|^{-1}$. This specific result for linear ϕ_i is the so-called MacMahon Master Theorem. To apply this to Dixon's problem of evaluating $S (**)$ we note that

$$\left(1 - \frac{x}{y}\right)^n \left(1 - \frac{y}{z}\right)^n \left(1 - \frac{z}{x}\right)^n$$

$$= \sum_{0 \leq i, j, k \leq n} (-1)^{i+j+k} C_i^n C_j^n C_k^n x^{i-k} y^{j-i} z^{k-j}.$$

The term independent of x, y, z is when $i = j = k$, which is S . Thus

$$S = [x^0 y^0 z^0] \left\{ \left(1 - \frac{x}{y}\right) \left(1 - \frac{y}{z}\right) \left(1 - \frac{z}{x}\right) \right\}^n \\ = [x^n y^n z^n] \{(y-x)(z-y)(x-z)\}^n$$

Now $m = 3$, $\phi_1 = z - y$, $\phi_2 = x - z$, $\phi_3 = y - x$, and

$$A = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} \partial\phi_i \\ \partial x_j \end{pmatrix}$$

where $(x_1, x_2, x_3) = (x, y, z)$. Then $S = [x^n y^n z^n] \{|I - XA|^{-1}\}$. Now

$$|I - XA| = \begin{vmatrix} 1 & x & -x \\ -y & 1 & y \\ z & -z & 1 \end{vmatrix} = 1 + yz + xy + xyz - xyz + zx \\ = 1 + xy + yz + zx,$$

$$\text{so } S = [x^n y^n z^n] (1 + xy + yz + zx)^{-1} \\ = [x^n y^n z^n] \sum_{\alpha, \beta, \gamma \geq 0} (-1)^{\alpha+\beta+\gamma} \frac{(\alpha+\beta+\gamma)!}{\alpha! \beta! \gamma!} x^{\alpha+\gamma} y^{\alpha+\beta} z^{\beta+\gamma}.$$

Therefore we must have $\alpha + \gamma = \alpha + \beta = \beta + \gamma = n$ or $\alpha = \beta = \gamma = n/2$. However α, β, γ are integers, which fact forces $n = 2m$ say, and $\alpha + \beta + \gamma = 3m$. So $S = \frac{(-1)^m (3m)!}{(m!)^3}$ as required.

Third Proof

This uses the method of WZ pairs [7, p.120ff]. The idea of a WZ pair is as follows: To prove the identity

$$\sum_k A(n, k) = f(n) \quad \text{say, for } n = 0, 1, 2, \dots \quad (\dagger)$$

(where the range of k may be from $-\infty$ to ∞) is equivalent to showing

$$\sum_k \frac{A(n, k)}{f(n)} = 1 \quad \text{for } n = 0, 1, 2, \dots \quad (3)$$

or letting $F(n, k) = A(n, k)/f(n)$ we can write (3) as

$$\sum_k F(n, k) = 1,$$

which is now independent of n . Replacing n by $n + 1$, it follows that

$$\sum_k \{F(n+1, k) - F(n, k)\} = 0 \quad \text{for } n \geq 0. \quad (4)$$

Suppose there exists a "nice" function $G(n, k)$ with the property that

$$F(n+1, k) - F(n, k) = G(n, k+1) - G(n, k), \quad (5)$$

then the series in (4) results in the telescoping of $G(n, k)$, to wit

$$\sum_{k=-L}^{+M} \{F(n+1, k) - F(n, k)\} = \sum_{k=-L}^{+M} \{G(n, k+1) - G(n, k)\} \\ = G(n, M+1) - G(n, -L).$$

Let us further require

$$\lim_{k \rightarrow \pm\infty} G(n, k) = 0, \quad (6)$$

then the identity $(\dagger) \sum_k A(n, k) = f(n)$ is proved.

It transpires that for a wide class of identities there are such "nice" functions $G(n, k)$ of the form $R(n, k)F(n, k-1)$ — where $R(n, k)$ is a rational function of n and k — and (\dagger) is proved by exhibiting F and R . The F and G obtained thus are referred to as a WZ pair if the conditions (5) and (6) hold. This procedure enables one to use symbolic manipulation packages to carry out

the steps by computer, and thus prove identity (†). In particular, Dixon's identity in the generalization of Fjelsted:

$$\sum_k (-1)^k \binom{n+b}{n+k} \binom{b+c}{b+k} \binom{c+n}{c+k} = \frac{(n+b+c)!}{n!b!c!}$$

is proved firstly by taking

$$R(n, k) = \frac{(c+1-k)(b+1-k)}{2(n+k)(n+b+c+1)}$$

thus $G(n, k) = R(n, k)F(n, k-1)$, and secondly by verifying equations (5) and (6) — for which laborious exercise it would be advisable to avail oneself of Macsyma say.

In general there are few known identities involving sums of products of several binomial coefficients. A spectacular generalization of Dixon's beautiful identity is given by equation 5.31 on p.171 of [5] which must surely be the *non plus ultra* of the species.

References

- [1] A. C. Dixon, *Summation of a certain series*, Proc. London Math. Soc. **35** (1903), 284-289.
- [2] J. E. Fjelsted, *A generalization of Dixon's formula*, Math. Scandinavica **2** (1954), 46-48.
- [3] I. P. Goulden and D. M. Jackson, *Combinatorial enumeration*. Wiley, 1983.
- [4] R. L. Graham, D. E. Knuth and O. Patashnik, *Concrete mathematics*. Addison-Wesley, 1989.
- [5] E. T. Whittaker, *Alfred Cardew Dixon*, J. London Math. Soc. **12** (1937), 145-154.
- [6] H. S. Wilf, *Generatingfunctionology*. Academic Press, 1990.

James Ward,
University College,
Galway.

SOME GROUPS OF EXPONENT p

J. D. Reid

§1 Introduction.

By the *exponent* of a (finite) group G is meant the least common multiple of the orders of the elements of G . It is a well known elementary exercise that groups of exponent 2 are abelian; and all groups of order p^2 , p a prime, are abelian. On the other hand there are examples of non-abelian groups of exponent p ($p > 2$) and order p^3 , or p^4 , that go back to Burnside, at least (e.g. [1]). Taking a direct product of a non-abelian group of order p^3 , for example, with an elementary abelian p -group of order p^n will, of course, give an example of a non-abelian group of exponent p and of arbitrarily large finite cardinality. However as an example of a non-abelian group of exponent p such a group offers little more than its non-abelian direct factor.

Our interest in examples of such groups was stimulated by questions of W. W. Comfort. We present here a simple construction of an infinite class of non-trivial (i.e. non-abelian and indecomposable) groups of exponent p , $p > 2$.

Observe that to say that a group G is abelian is to say that it is equal to its centre, $z(G)$, so that the larger the centre of G the more abelian, in a sense, is G . Similarly G is abelian if and only if its derived group G' is trivial so that the smaller the derived group, the more abelian is G . It may happen that $z(G)$ is contained in G' in which case G has no hope of being abelian: the larger the centre in G the larger the derived group, the smaller the derived group the smaller the centre. Hopes for commutativity are frustrated just in proportion to their strength. For the purposes of this discussion we encapsulate this idea in the

Definition 1.1 A group G is *inherently non-abelian* if its centre is contained in its derived group.

Note that if G is a direct product of its subgroups K and M then G is inherently non-abelian if and only if both K and M are inherently non-abelian, so that the indecomposable ones are the ones of interest. We observe also that the group consisting of the identity element alone is the only abelian inherently non-abelian group.

Our object is to prove the following

Theorem 1.2. For each odd prime p and every integer $s > 0$ there exists an indecomposable inherently non-abelian group G of exponent p and order $p^{s(p-1)+1}$.

There are several ways to go about this. We have chosen what seems to us a fairly natural and conceptual one. For another more elementary but perhaps slightly ad hoc approach, see the remarks at the end of the paper.

§2 Finite Fields.

We recall some facts about finite fields. Let L be a field of cardinality p^s and let F be the extension of L of degree p . Then F is a Galois extension of L with cyclic Galois group which we denote by Γ . Let σ be a generator of Γ .

We will frequently think of elements of Γ as being simply linear transformations in F as vector space over L , so that we may add them together as well as multiply them. For example, in the polynomial ring $L[t]$, t an indeterminate, we have

$$(1-t)(1+t+\cdots+t^{p-1}) = 1-t^p = (1-t)^p = (1-t)(1-t)^{p-1}$$

so that $1+t+\cdots+t^{p-1} = (1-t)^{p-1}$. Hence for the automorphism σ , $1+\sigma+\cdots+\sigma^{p-1} = (1-\sigma)^{p-1}$ and therefore the trace map of F over L is given by $(1-\sigma)^{p-1}$. Since the trace map has image L , we have

$$L = (1-\sigma)^{p-1}F.$$

We write H for the set of elements of trace 0. This is an L subspace of F of codimension 1 and is mapped onto itself by

the automorphisms in Γ . Our formula for the trace map shows that $(1-\sigma)F \subseteq H$ since $(1-\sigma)^p = 1-\sigma^p = 0$. On the other hand the trace map itself is not zero so $(1-\sigma)^k$ is non-zero for all k , $0 \leq k \leq p-1$. It follows that $(1-\sigma)^{k+1}F$ is contained properly in $(1-\sigma)^kF$ for all k , $0 \leq k \leq p-1$. Since there are p such subspaces of F and F has dimension p over L , it follows that $(1-\sigma)^{k+1}F$ has codimension 1 in $(1-\sigma)^kF$. In particular $(1-\sigma)F$ has codimension 1 in F and the inclusions $(1-\sigma)F \subseteq H \subset F$ yield

$$H = (1-\sigma)F. \quad (1)$$

This is a special case of a general fact about cyclic extensions. See for example [2].

§3 Basic Properties.

We define G to be the semi-direct product of the additive group H and the multiplicative group Γ , noting that elements of Γ induce automorphisms on H . Thus G is the cartesian product of H and Γ with multiplication defined by

$$(x, \rho)(y, \tau) = (x + \rho(y), \rho\tau), x, y \in H; \rho, \tau \in \Gamma.$$

The identity element of G is $(0, 1)$ and inverses are given by $(x, \rho)^{-1} = (-\rho^{-1}(x), \rho^{-1})$. We have

$$(x, \rho)(y, \tau)(x, \rho)^{-1} = ((1-\tau)x + \rho(y), \tau) \quad (2)$$

and

$$(x, \rho)(y, \tau)(x, \rho)^{-1}(y, \tau)^{-1} = ((1-\tau)x - (1-\rho)y, 1) \quad (3)$$

for $x, y \in H$ and $\rho, \tau \in \Gamma$.

Observe that $\{(x, 1) : x \in H\}$ is a normal subgroup of G , that contains G' by (3) above, and is isomorphic to H under the map $x \mapsto (x, 1)$. We will frequently identify H and its subgroups with this subgroup of G and its subgroups. For example L will be identified with the subgroup $\{(x, 1) : x \in L\}$ of G . Similarly, $\{(0, \rho) : \rho \in \Gamma\}$ is a subgroup of G isomorphic to

Γ and will occasionally be denoted by Γ as well. This should cause no confusion, but can be useful. For example the subgroup $\{(x, \rho) : x \in L, \rho \in \Gamma\}$ of G is isomorphic to the direct product of L and Γ , since Γ acts trivially on L , and by our conventions we would denote it by $L \times \Gamma$, or $L\Gamma$.

A straightforward induction yields a power formula: for $x \in H, \rho \in \Gamma$,

$$(x, \rho)^m = \left(\sum_{j=0}^{m-1} \rho^j(x), \rho^m \right).$$

Since Γ has exponent p and H consists of elements of trace 0 this formula shows that $(x, \rho)^p = (0, 1)$ if $\rho \neq 1$, while $(x, 1)^p = (px, 1) = (0, 1)$ since we are in characteristic p . Hence

Corollary 3.1. G has exponent p .

In the proposition below we use the convention of identifying subgroups of H with certain subgroups of G . Thus by $(1 - \sigma)H$ here we mean $\{((1 - \sigma)x, 1) : x \in H\}$.

Proposition 3.2. The derived group, G' , of G equals $(1 - \sigma)H$.

Proof: For any $\rho, \tau \in \Gamma$ and $x \in H$, we have

$$((1 - \tau)x, 1) = (x, \rho)(\theta, \tau)(x, \rho)^{-1}(0, \tau)^{-1} \quad (4)$$

and by (3)

$$\begin{aligned} (x, \rho)(y, \tau)(x, \rho)^{-1}(y, \tau)^{-1} &= ((1 - \tau)x - (1 - \rho)y, 1) \\ &= ((1 - \tau)x, 1)((1 - \rho)y, 1)^{-1} \end{aligned}$$

so that the elements $((1 - \tau)x, 1), x \in H, \tau \in \Gamma$, generate G' . Any $\tau \in \Gamma$ has the form σ^k for suitable $k, 0 \leq k < p$, so

$$1 - \tau = 1 - \sigma^k = (1 - \sigma)(1 + \sigma + \dots + \sigma^{k-1}).$$

It follows that $((1 - \tau)x, 1) \in \{((1 - \sigma)w, 1) : w \in H\} = (1 - \sigma)H$ so that $G' \subseteq (1 - \sigma)H$.

On the other hand, it is clear from (4) that $(1 - \sigma)H \subseteq G'$.

Corollary 3.3. G' is abelian.

The following simple fact is obvious but we state it explicitly for emphasis since its corollary lies at the heart of the non-commutativity of G .

Proposition 3.4. The set H of elements of trace 0 in the extension F of L has cardinality $p^{s(p-1)}$.

Proof: F has cardinality p^{ps} and the image, L , of the trace map has cardinality p^s . Hence the kernel, H , of the trace map has cardinality $\frac{p^{ps}}{p^s} = p^{s(p-1)}$.

Corollary 3.5. H and L are equal if and only if $p = 2$.

§4 Main Results.

We now restrict p to be an odd prime. By the Corollary above, L is then a proper subset of H and since F has degree p over L , we see that H contains a generator of F out of L . As a consequence, the only automorphism of F over L that leaves H elementwise fixed is the identity map. This fact plays a large role in what follows. For example

Proposition 4.1. The centre, $z(G)$, of G is $\{(x, 1) : x \in L\}$.

Proof: From (2), $(x, \rho) \in z(G)$ if and only if $((1 - \tau)x + \rho(y), \tau) = (y, \tau)$ for all $y \in H, \tau \in \Gamma$. Taking $\tau = 1$, we obtain $\rho(y) = y$ for all y in H . Hence $\rho = 1$. Now taking $y = 0$ and τ arbitrary in Γ we have $x = \tau(x)$ for all τ so $x \in L$ as required. Conversely it is clear from (2) that $(x, 1) \in z(G)$ for all $x \in L$.

Theorem 4.2. The group G is inherently non-abelian, i.e. $z(G) \subseteq G'$.

Proof: We have determined both the centre and the derived group. Each is a subgroup of H , identified with its canonical image in G . We have already observed that $(1 - \sigma)^{p-1}F = L$. Hence we have

$$z(G) = L = (1 - \sigma)^{p-1}F \subseteq (1 - \sigma)^2F = (1 - \sigma)H = G'$$

by Proposition 4.1, Proposition 3.2 and (1). Note that this inclusion uses again the fact that $p \geq 3$, i.e. $p - 1 \geq 2$.

Another qualitative indication of the lack of commutativity of G is the fact that the centralizer of each element is as small as can be—i.e. elements of G commute only with the obvious, such as their own powers, elements of the centre, etc. It is not hard to identify these centralizers (as we did in our original proof) but we only need the following to complete our discussion. We are indebted to the referee for the elegant treatment of case (iii).

Proposition 4.3. *If $g \notin z(G)$ then the centralizer of g in G is abelian.*

Proof: The non-central elements of G have the form $g = (x, \rho)$, with $x \notin L$ or $\rho \neq 1$. We consider three cases.

(i) $x \notin L, \rho = 1$. Here g is in H which is a commutative maximal subgroup of the non-commutative group G , so $C_G(g) = H$ in this case.

(ii) $x = 0, \rho \neq 1$. By (2), $(y, \tau) \in C_G(g)$ if and only if

$$(y, \tau) = (0, \rho)(y, \tau)(0, \rho)^{-1} = (\rho(y), \tau).$$

Thus $C_G(g) = \{(y, \tau) : \rho(y) = y\} = \{(y, \tau) : y \in L\}$ since $\rho \neq 1$ and Γ has ρ as generator. Here then $C_G(g)$ is the direct product of L and Γ .

(iii) It remains to deal with the case $x \neq 0, \rho \neq 1$. The map $\alpha : G \rightarrow G$ defined by

$$(y, \rho^k)^\alpha = (y + \sum_{j=0}^{k-1} \rho^j(x), \rho^k), (y \in H, k \geq 0)$$

preserves the multiplication in G , hence is an automorphism. Putting $h = (0, \rho)$, we have $C_G(g) = C_G(h^\alpha) = C_G(h)^\alpha$, which is abelian by (ii).

To complete the discussion, we have

Theorem 4.4. *G is indecomposable.*

Proof: Suppose that $G = K \times M$, direct product. Then M is contained in the centralizer of each element of K , and K is not contained in the centralizer of each element of M . If K is not contained in $z(G)$, then M is commutative by Proposition 4.3. Similarly for M . Hence we may assume that K , say, is contained in the centre of G .

Now $z(G) = z(K) \times z(M) = K \times z(M)$ since K is commutative. We also have $G' = K' \times M' = M'$ since K is commutative. But this gives $K \subseteq z(G) \subseteq G' = M' \subseteq M$, and since $K \cap M$ is trivial, K is trivial.

Our main result has now been established. The group G is an indecomposable, inherently non-abelian group of exponent p and has order $p^{s(p-1)+1}$.

§5 Remarks. As another approach to this subject one could view a p -dimensional vector space V over the field L of cardinality p^s as an $L[t]$ module via the linear transformation given, in some basis, by the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 1 & 0 & \dots & 0 \\ \cdot & \cdot & \cdot & \cdot & \dots & \cdot \\ 0 & 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

Then V would play the role of F in our previous discussion, the kernel of ΣA^j that of H , and in place of σ one would use A .

The groups constructed above have many special properties, perhaps even enough that they admit an easy characterization. We do not pursue that question here though it might be of interest to point out that the subgroup H which plays such a prominent role has a group-theoretic characterization, at least for $p > 3$. For such p , $H = C_G(G')$.



References

- [1] W. Burnside, *Introduction to Finite Groups* (2nd Edition). Dover: New York, 1955.
- [2] Nathan Jacobson, *Basic Algebra I*. Freeman: New York, 1985.

J. D. Reid,
 Department of Mathematics,
 Wesleyan University,
 Middletown,
 Connecticut 06459,
 USA.

Book Review

Mathematics and its History
 (Undergraduate Texts in Mathematics)

J. Stilwell
 Springer-Verlag, 1989,
 ISBN 3 540 96981 0.

Reviewed by James Ward

John Stilwell, in addition to his original contributions in mathematics, is the translator of Serre's *Trees* (Springer-Verlag), and the author of *Classical Topology and Combinatorial Group Theory*. His experience as a writer on mathematics shows to good advantage in the volume under review.

Proceeding from the observation (probably all too true in many universities) that students are taught Algebra, Calculus, Group Theory, Topology, Measure Theory etc, and are taught little of the connexions between these areas, the author's aim is to combine the ingredients of Mathematics, using History as a leavening agent; the result is very appetizing indeed!

This perspective differs from that of well known books on the subject, such as the works of Boyer and Struik — to name but two — who are more concerned with tracing the evolution of mathematical ideas; also they aspire to produce a complete account of the history of the subject (Struik being telegraphic in style but, given its length, remarkably complete; Boyer is very comprehensive).

In Stilwell's book, the presentation of material in each chapter is followed by a section of Biographical Notes, which in most cases includes illustrations of the mathematicians mentioned in the chapter. This is very useful for a lecturer seeking last-minute



information on say Pascal or Fermat or Descartes! (One may not infer that this is an empirical observation of mine.)

The topics covered in the book are: 1. The Theorem of Pythagoras; 2. Greek Geometry; 3. Greek Number Theory; 4. Infinity in Greek Mathematics (a wise 2nd year student once wrote "Euclid proved the number of primes is infinite, and nobody knows how many there are"); 5. Polynomial Equations; 6. Analytic Geometry; 7. Projective Geometry; 8. Calculus; 9. Infinite Series; 10. The Revival of Number Theory; 11. Elliptic Functions; 12. Mechanics; 13. Complex Numbers in Algebra; 14. Complex Numbers and Curves; 15. Complex Numbers and Functions; 16. Differential Geometry; 17. Non-Euclidean Geometry; 18. Group Theory; 19. Topology; 20. Sets, Logic and Computation.

In his Preface, the author pleads an excuse for not including Lie Groups or Functional Analysis, and also for not mentioning Probability Theory. The choice of topics is nonetheless very wide, and is an ideal for the breadth of knowledge of a mathematics graduate. Some of the material is quite sophisticated, but the author's exposition is very clear and concise (in particular in Chapter 11, whose topic does not commonly feature in undergraduate curricula, and which would serve as an ideal background to a post-graduate course on that area).

The last chapter is particularly welcome (not all universities offer a course on Set Theory or Mathematical Logic) especially since rapid developments in Computer Science are impinging on mathematics curricula. I think it is worthwhile listing the sections in this chapter: An Explanation, Sets, Measure, The Diagonal Argument, Logic and Gödel's Theorem, Provability and Truth, Biographical Notes: Gödel. I might add that there is a curious aperçu of Gödel's character on pp. 28–29 of *Conversations* by Primo Levi and Tullio Regge (the late celebrated Italian author and chemist, and renowned physicist respectively).

As one has come to expect from Springer-Verlag, the book is extremely well produced. I failed to spot any misprints of consequence, although there is an egregious blunder in the copyright statement caused by the omission of the word "not" — this is corrected in an erratum. This adds a certain piquancy to the book.



I recommend very highly *Mathematics and its History*. It was a pleasure to read it, and be reminded of the beauty and purity of mathematics as a human endeavour.

James Ward,
University College,
Galway.

INSTRUCTIONS TO AUTHORS

The Bulletin is typeset with \TeX . Authors should if possible submit articles to the Bulletin as \TeX input files; if this is not possible typescripts will be accepted. Manuscripts are not acceptable.

Articles prepared with \TeX

Though Plain \TeX files are preferred, authors may use \AMS-TeX or \LaTeX . A layout file, `imsform.tex`, is available from the Society on request or can be got from Wayne Sullivan's MATHDEP. The *MISTRESS* package for dealing with citations and reference lists is also available to contributors. Instructions on the use of these is contained in the article on *Directions in Typesetting* in issue number 27, December 1991.

The \TeX file should be accompanied by any non-standard style or input files which have been used. Private macros, reference input files and both \METAFONT and \TeX source files for diagrams should also accompany submissions.

The input files can be transmitted to the Editor either on an IBM or Macintosh diskette, or by electronic mail to the following Bitnet or EARN address:

`MATDARK@BODKIN.UCG.IE`

Two printed copies of the article should also be sent to the Editor.

Other Articles

Authors who prepare their articles with word processors can expedite the typesetting of their articles by submitting an ASCII input file as well as the printed copies of the article.

Typed manuscripts should be double-spaced, with wide margins, on numbered pages. Commencement of paragraphs should be clearly indicated. Hand-written symbols should be clear and unambiguous. Illustrations should be carefully prepared on separate sheets in black ink. Two copies of each illustration should be submitted: one with lettering added, the other without lettering. Two copies of the manuscript should be sent to the Editor.